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A CAUTIONARY TALE ABOUT RABBITS AND MOLES AND CHOICES.

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A. CLOSING.

This cautionary tale of lost and gained opportunities starts many years ago. The story concerns Alfredo, a boy who is good at mathematics. To find out more about him, we can use our powers of imagery to travel the necessary distance to surreptitiously view him in his school mathematics classroom. Look closely. The desks in the classroom are laid out in rows facing the teacher who is at the front talking to the class. Our hero is sitting in the second row from the door, third desk from the front. He's rather restless at the moment, doing his best to let the teacher and the class knows that he is the top pupil in the class and understands the work already.

He is good at maths and usually comes top in class tests and examinations. He has a natural ability as well as a well-developed sense of survival in the classroom. He has karnt many things over the years - such as always starting with teacher's method and then improving on it later - and such as refining his methods so that he's the fastest in the class. He also knows that it would be foolish to admit to any hesitation in the class. His peers are like a pack of wolves ready to pounce with roars of cruel laughter at any mistake he makes. He also knows that wrong answers are there to be scorned, so wherever possible he waits until he knows that he is correct before committing himself.

Wait a minute. The teacher has finished talking and is giving them a spot test. Alfredo, who has been becoming increasingly tense, gets his workbook out quickly and attacks the test - silently and competitively. He knows the rules so he works at topspeed, making sure that all rough work is done on another sheet of paper, so that all that can be seen is the very neat, well-presented final version. He proudly finishes first with a noisy flourish. Only now, under a mask of nonchalance, does he surreptitiously start checking his answers to make sure he hasn't made a fool of himself by making mistakes. He is relieved to find that he hasn't and feels secure - once again he has been able to demonstrate his superiority in the classroom. He is immediately rewarded by being asked to leave his desk and explain the work to one of the other's who has failed the test. Alfredo has learnt that maths is an abstract subject in which there are a set of procedures to be learnt as simply as possible. These procedures can be refined and extended to allow complex and 'trick' questions to be answered.

But if we look closely, we can see that all is not well with Alfredo. His satisfaction after his success does not seem to really last for any length of time. Rather than wait and savoir the feel of the achievement, he's off on his next task - looking for his next fix of success. When we follow him onto the playground at break, we see Alfredo walking off on his own - finding it difficult to engage with his classmates. Out of the classroom his status is removed and he feels exposed and inadequate. And at home he doesn't read books on mathematics.

His family does not have our advantage and cannot see him at work and play. They are only able proudly to receive the school reports filled with the conditioning words of "Excellent" and "Outstanding Work". Sometimes they tease each other as to which of them Alfredo really takes after.

We can leave this classroom scene of the past, secure in the knowledge that Alfredo's future has been decided. He leaves school as the top mathematician gaining distinctions in both mathematics subjects. He applies the same tried and tested methods successfully to his University courses and graduates to become a mathematics teacher. And he continues the cycle by ensuring that his class does extremely well in tests and examinations as he coaches them enthusiastically in quick and direct methods of solving problems. But he still doesn't read mathematics for fun!

B. OPENING.

And that would normally have been the end of the story. However, one day, a very wise man, who also happens to be a mathematician, asks Alfredo some very important questions. "Tell me, what exactly is algebra to you?" " Here is a problem - how would YOU go about solving it?" " What are you FEELING as you attempt to solve this problem?" " What do YOU think?" The questions are puzzling - there doesn't seem to be a right answer. Certainly the wise man gives no indication of expecting any particular answer - and generally each response is greeted with the same amount of interest.

In this way a journey into a different world begins for Alfredo. It's a world where his security in algorithms and compartmentalised knowledge is turned upside down. He discovers that Euclidean Geometry isn't meant to be static as it's presented in textbooks. Lines can move if YOU want them to, and you can see it using film and acetate. And there are other geometries!! Including one in which points and lines can be interchanged without making the theorems invalid, and which links directly into nature and art. He reads bewildering statements such as "Geometry is the awareness of imagery and Algebra is the dynamics of imagery" and starts thinking???

And he soon finds that this imagery is a power which he used to possess. A power which school has discouraged. And he welcomes it back! ...What would happen if he became a point on a graph moving a set distance from two poles? ...What would happen if he lived in a 2 dimensional world - and he finds a book called Flatland about such a person.

And he starts discovering how he chooses to learn - and it's nothing like his school experience. He likes to play and mathematise with concrete objects. He likes to share his work at times with others as they struggle at a problem - and then he drifts away when he is ready. But he learns that others are not stupid or inferior to him, and he experiences that the strangest people come up with wonderfully helpful insights at different times if only his ears are open to hear them. And when he discovers something for himself he feels excited and alive. He wants to shout aloud with an inner joy. He's learnt that it's impossible to share it with others yet, because they can't experience what he's experienced because they are not where he was. It's his joy and he has learnt that he can't reproduce that insight for anyone else. So he goes inwards and enjoys the quiet glow. He repeats his discovery over and over again, marveling at his own brilliance. He will return to that same discovery many times in his lifetime to re-live that moment. And the thrill will never go away. The pleasure and the insight remain forever, because this knowledge is not out there - it's in him. And only when he's ready does he move on with the certain knowledge that a problem is never complete since wise men can always ask another question...what would happen if...?

He discovers that this is how real mathematicians work. He reads about their lives and finds drama, controversy and passion. Their notebooks certainly do not have nice reat proofs with the equal signs written carefully and neatly underneath each other. (He wonders how many marks teachers

would have had subtracted for their untidy work). And he finds that several of them were failures at school. He wonders if Fermat had a wicked sense of humour!

And most of all he learns to embrace confusion and chaos. How wonderful it is to be ignorant. He welcomes wrong answers as an opportunity to explore his boundaries and learn. He takes pleasure in these wrong answers and, slowly and joyfully, unpeals the many layers of his ignorance.

And he does it his way - fully acknowledging the contributions of others, but making the decisive choices himself. He is in control, because he is the authority! Only he can know what he knows at any moment and the way that he knows it. Alfredo is alive and feels he is experiencing learning and mathematics for the first time in his life!

As he travels on this new journey, he becomes aware of an enormous anger building up inside him. Why has this wonderful world of mathematics and learning been hidden from him? Why did no one tell him that he had the power to make his own decisions and choose his own learning? How dare THEY hide all this from him and manipulate him into conforming to the system? How dare they chain him to his desk and programme his life for him! He feels cheated and betrayed!

And as the anger starts to move away, he starts to think about how things might have been..... The decisions he would have changedand all his mistakes come flooding up before his eyes - and he starts to mourn his previous lack of insight and choice.

And then suddenly he starts to feel uncomfortable. At what stage are you ready to start making decisions, he wonders....He has begun to believe that no-one can teach you something you don't already know. What if a corollary is also true....."No-one can stop you taking responsibility for your own decisions if you know you are ready to take that responsibility?"....Maybe it is only now that he has become ready to assert himself.

He starts to reflect on all the time that he would waste if he carried on blaming THEM and the SYSTEM. He decides that they have to worry about their own actions. He can only take responsibility for himself. So with a deep sigh he steps out into the garden and continues to explore.

C. THE CYCLE CONTINUES.

Is this the happy ending? Unfortunately real change does not come so easily and the cycle continues. Alfredo has a son and that son, aged 8, like all children goes to school. At school he does well in class at both sport and work. In fact he does very well and comes first in class. He becomes obsessed with winning and starts getting tense about going to school because he wants to win and hasn't been beaten yet.....

Fortunately Alfredo's son has a wise teacher who comes into his life at a much earlier stage than was Alfredo's lot. This wise teacher tells Alfredo's son the following story in his end of year school report.

"Once a rabbit and a mole decided to have a race. Of course rabbit knew he would win and while mole was burrowing underground, doing his utmost to reach the finishing post, rabbit took time to visit Farmer Simon's fields to dine on some fresh herbs. He was chased and almost caught. Then he slept beneath shady trees, gently enjoying the balmy wind and smelling the sweet moist earth and perfumed flowers. All the while mole burrowed on underground. Then rabbit paid a visit to some squirrels and was chased by hounds. He escaped by jumping into a river, where he met two frogs and ended up listening to their river stories. All the while mole burrowed on blindly underground. Rabbit became aware of the time and ran to the finishing line under the most beautiful sunset skies. Alas, he was too late, there was the mole, panting with exhaustion. Rabbit had not won, but what a difference in experience was his race!!"

Time will tell if this story was offered at the right time.

D. CONCLUSION.

Much has changed since Alfredo was at school. New syllabi have resulted in new topics being taught. Exciting books have been written which make mathematics and mathematicians more accessible. New teaching methods have come and gone on a tide of enthusiasm. Yet the same if not more rigidity can be found in the system. The desks generally still face the front in rows, and the teachers are kept under enormous pressure. They cope and make the required ticks in their record books by talking most of the time. They feel they have little choice in the matter, because they are controlled by inspectors and tests, and by their nature these tests define the work to be covered. There is never enough time to explore. Industry and Universities put pressure on the system by using maths results as a major selection criterion. Parents know this reality and do what they can to assist their child get good results. And students soak in this message and ask "is it for marks" before deciding whether to listen. And the moles rule!

We are living in a time of change. Why should this change only take place out there? Why don't you take this opportunity to stand back for a moment and reflect on your experience of the race? Is it similar to that of rabbit or of mole? Ask yourself some questions. "Who controls my learning? What are my personal powers? What excites me about mathematics? Is it its beauty and elegance and poetry, or is it merely the thrill of success and winning." And if you are a teacher, think about the environment you create in your classroom and the messages you give to your students. "Who controls your teaching? What are your personal powers?"

So this tale is for all who feel that, together with mole, they have been locked into Flatland by our schooling system. As long as we feel that the power and the knowledge lie outside of us, we will remain trapped in Flatland. If the time is right for us to know our powers, we have the possibility of transforming our experience of teaching, learning and mathematics. The choice is ours!