RE-COGNISING
TEACHING AND LEARNING
IN AN
AD MATHEMATICS PROGRAMME.

by

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Declaration:

This work has not been previously submitted in whole, or in part, for the award of any degree. It is my own work. Each significant contribution to, and quotation in this dissertation from the work, or works, of other people has been attributed, and has been cited and referenced.

Signature: Date: 14 February 2003

ABSTRACT

This dissertation deals with me and my practice as a mathematics teacher in a high school in Cape Town. Through the lens of my teaching in an Ad (advanced, accelerated and enriched) mathematics programme I re-cognise the teaching and learning opportunities that have presented themselves over a three year period. I attempt an understanding of these opportunities through building a theoretical research space that blends quantitative and qualitative aspects by drawing on the new holistic theories of cognition inherent in enactivism.

This is a piece of work which attempts to foreground my voice as a teacher and draws data, both quantitative and qualitative from my practice in a continual feedback loop of questioning leading to data, leading to interpretation, leading to action, leading to questioning.

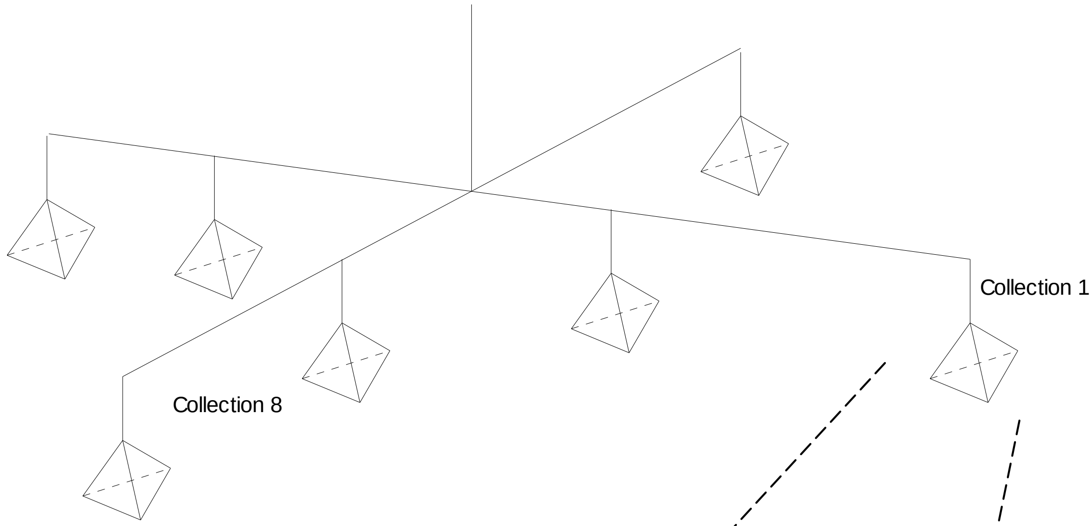
I lean heavily on the research method known as the Discipline of Noticing which attempts to give practising teachers a means of interrogating their practice and of building theory from within that practice.

The statistical technique of regression discontinuity analysis is employed to contrast the results of those who have been in the Ad programme with those who have been in regular classes. Threads from pupil and teacher reflections are used to investigate more deeply the questions raised by this quantitative data. Video material and short-response questionnaires are used to open up perceptions by my pupils of my teaching in a number of classes.

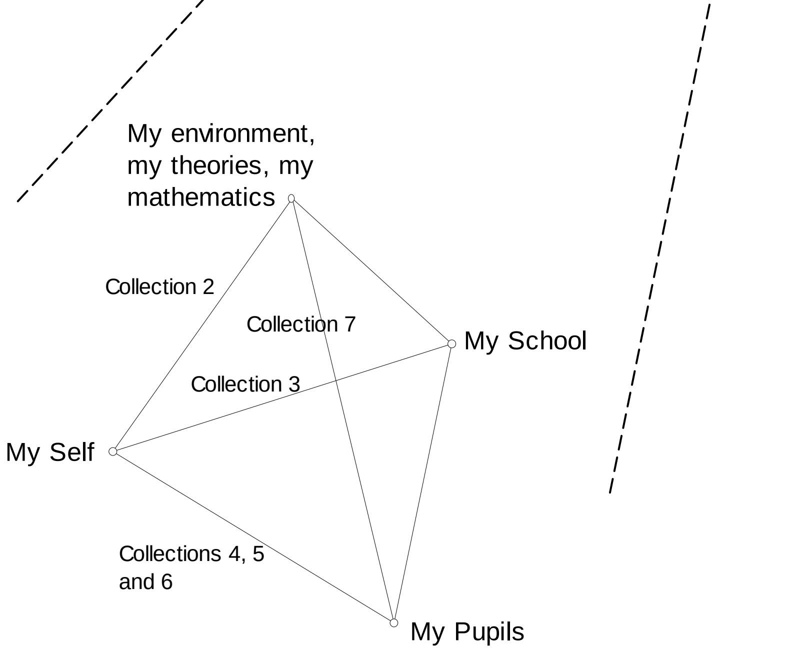
CONTENTS

OVERVIEW

The study that I have attempted, by its nature of being about my practice, is complex. I would thus like to provide an overall map of the space that my work occupies and the paths through that space, to assist you in your reading.



My academic space occupies a tetrahedron hanging from a mobile with other tetrahedra. That tetrahedron has four points being the four extremes of my space – self, pupils, school and environment / theories / mathematics. It is important to note that there is no path to my self, my pupils and my school, except via my environment and current theoretical and mathematical paradigms.



As you enter the work, my story will unfold. The constraints of writing such a story, force one into a linear pattern, but the experience has been anything but linear. This schematic will hopefully assist you to understand both the complex and complicated (Davis and Sumara, 1997) manner in which the story has grown for me.

As such I have grouped my research not into chapters, but into collections of thoughts and actions that share a common space or path. It makes no sense for me to present a standard format of introduction, literature review, methodology, results, discussion and conclusion, when these concepts have become so mutually intertwined, continually feeding back on one another. To separate them would destroy even further the glorious cyclic and chaotic nature of research.

Collection 1 of my thoughts and actions takes us into my space.
Collection 2 occupies the axis linking my self to my environment / theories / mathematics, giving a brief introduction to the influences on my current thinking.
Collection 3 occupies the axis linking self to school and is a motivation for and description of the Ad mathematics programmes within which I have worked.
Collections 4, 5 and 6 occupy the axis of self and pupils and consist of reflections on interactions, both in and outside the class environment, between myself and those I teach. Collection 4 is weighted towards the pupils, collection 5 towards my self and collection 6 is at equilibrium.
Collection 7 occupies the space created by the whole tetrahedron and is an attempt to draw some conclusions from, and ask some questions about the data that make up the dissertation.
Collection 8 occupies the space of others and is my list of references and acknowledgements.

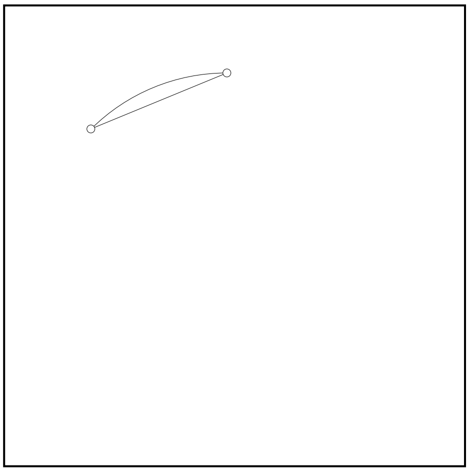
It is within this space that I attempt to position my teaching. If I position my teaching too close to self, to pupils, to school or to environment, then the space becomes distorted and off balance. I can choose to make my teaching a copy of someone else, but I am then forced to position myself outside of my space. It is my ideal to centre my teaching, mathematically in line with all four elements. This dissertation is about this balancing act.

COLLECTION 1
OF MY THOUGHTS AND ACTIONS:
A WAY IN.

A section in which I explore a setting for my dissertation and set the scene for my further thoughts and actions.

"They had forgotten much, but did not know it. What was beyond the walls of the city was no concern of theirs; it was something that had been shut out of their minds. Diaspar was all that existed, all that they needed, all that they could imagine. It mattered nothing to them that Man had once possessed the stars."
(Arthur C. Clarke in *The City and the Stars*, 1956: 9)

"Down how many roads among the stars must man propel himself in search of the final secret? The journey is difficult, immense, at times impossible, yet that will not deter some of us from attempting it . . . We have joined the caravan, you might say, at a certain point; we will travel as far as we can, but we cannot in one lifetime see all that we would like to see or to learn all that we hunger to know."
(Loren Eiseley in *The Immense Journey* (Buscaglia, 1982: 1))



In the futuristic world created by Arthur C. Clarke (1956) in his novel *The City and the Stars*, Clarke describes the city of Diaspar. A city set up to reproduce itself. No change is possible as humans live and relive their lives in a never-ending cycle generated by the memory banks of a computer. The comfort of the citizens is ensured by the removal of all the pain associated with change, the removal of all the pain associated with having to think outside the bounds of that which is accepted - that-which-has-always-been-done. Khedron, the jester, is part of the city and is meant to introduce small amounts of instability for the sole reason of ensuring that long-term stability is never challenged. Jeserac, the teacher, ensures that countless generations are raised as they have always been raised, so that they too retain the stability of the city.

But . . . .

Humanity cannot be held rigidly in certain ways of thinking and being. Alvin, a citizen of Diaspar, wonders what there is outside the city of Diaspar and dares to convert his **thoughts** into **actions**. By so doing he brings the city of Diaspar together with its conveniently forgotten neighbour of Lys, a city that seeks continual change. The coming together of different ideas is painful, but creates a space for both to become more whole than they were.

In a similar manner, research methods, some long established; some more recent additions to the landscape are currently redefining the concept of research in our post-modern society.

For the ancient Greek researchers and founders of the scientific method, a truth outside of human experience was that which was sought, a truth that did not rely on the senses:

> "This was the Pythagoreans' greatest contribution to civilisation – a way of achieving truth which is beyond the fallibility of human judgement."
>
> (Singh, 1997: 28)

For Rene Descartes it was a case of "*Cogito ergo sum*" ("I think therefore I am.") and the separation of researcher and researched continued; the separation of thought, emotion and body became entrenched.

There were, however, Alvins in the city who began the querying of things as they were. The very scientific tradition that gave birth to the separation, now gave birth to the siblings of quantum, chaos and complexity theories. Fritjof Capra was one of the voices that began to speak from the inside in a call for more unified thinking and presented this in books such as *The Tao of Physics* (Capra, 1975), *The Turning Point* (Capra, 1982), *Uncommon Wisdom* (Capra, 1988) and *The Web of Life* (Capra, 1996).

The ideas of chaos, complexity and deep ecological thinking were popularised (sometimes superficially) in movies such as *Mindwalk*, *Jurassic Park* and *Sliding Doors*.

Quoting the *I Ching*, Capra (1982: vii) paves the path for new thought:

> "After a time of decay comes the turning point . . . There is movement . . . The movement is natural, arising spontaneously. For this reason the transformation of the old becomes easy. The old is discarded and the new is introduced. Both measures accord with the time; therefore no harm results."

And so Descartes' separation moved towards the enactivist "to think is to know" and "all doing is knowing, and all knowing is doing" of Maturana and Varela (1987: 26). Davis (1996) extends enactivism to the mathematics teaching context and suggests that "every act is an act of cognition" (Davis, Sumara and Luce-Kapler, 2000) and suggests a moving of the concept of cognition away from 'to think' to 'to know'.

In my dissertation I try to combine many things:

- the thinking, the knowing and the being are inseparable
- the quantitative and qualitative tell overlapping parts of a full story.
- my reviewed literature, methodology, results and findings are continual loops feeding back on each other and are not disjoint entities.

- my self, my pupils, my school, my education system are intertwined – no one can be pulled from the equation without affecting the others.
- my teaching, my learning, their teaching and their learning are parts of a whole.

This dissertation is as much about my grappling with changing research methods as it is about my researching my teaching.

Jane (2001: 26), in reviewing the current deep ecology notions initiated by Capra, reinforces that "deep ecology is a holistic, ecological worldview where the world is regarded as an integrated whole."

This then is my whole story.

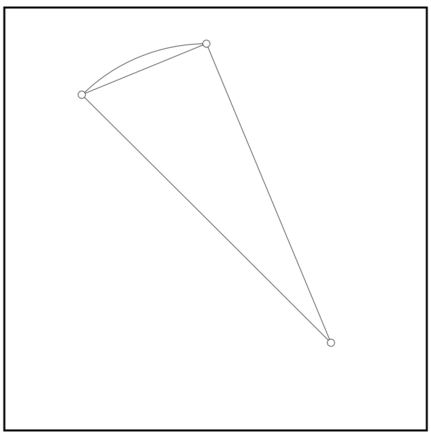
COLLECTION 2
OF MY THOUGHTS AND ACTIONS:
A DETOUR TO MYSELF

A section in which I sketch my education and the conflicts and experiences that have brought me to the current academic space that I occupy. I search for a way to combine the disparate parts of my education, utilising the positive of each discipline.

"You've always opposed us for wanting change," said the councillor of Lys, "and so far you have won. But I don't think the future lies with either of our groups now. Lys and Diaspar have both come to the end of an era, and we must make the best of it."
(Arthur C. Clarke in *The City and the Star, 1956*: 187)

"Those who investigate the phenomenon of life are as if lost in an inextricable jungle, in the midst of a magic forest, where countless trees unceasingly change their place and their shape. They are crushed under a mass of facts, which they can describe but are incapable of defining in algebraic equations . . . Man is an indivisible whole of extreme complexity. No simple representation of him can be obtained . . . In order to analyse ourselves we are obliged to seek the help of various techniques and, therefore to utilise several sciences."
(Carrel, 1946: 15-16)

"To deny the truth of our own experience in the scientific study of ourselves is not only unsatisfactory, it is to render the scientific study of ourselves without a subject matter."
(Varela, Thompson and Rosch, 1991: 13)



Learning for me started and continued for a long time in the acquisition metaphor mode described by Anna Sfard (quoted in Smith, 2001). In school and at university, my learning was about successfully producing the correct numbers and acquiring the needed pieces of paper – much in the realm of a “getting an education” mode as suggested by Gadamer (quoted in Breen, 2000). I studied to amass knowledge, filling my undergraduate years with as many courses as possible, still firmly trusting that one day I would be able to add my bit to the sum total of human knowledge. It was about an arithmetic progression of human knowledge, not a chaotic fractal unfolding.

Late in my first postgraduate course in the environmental sciences, I started to experience some dissatisfaction with the answers that I was deriving from masses of obscure meteorological data. There was a natural and interesting dichotomy set up in the Environmental and Geographical Sciences department between the physical geographers (who mostly followed a quantitative, mechanistic research paradigm) and the human geographers (who followed a qualitative, social sciences paradigm).

I was researching the physical realm and the data I was using seemed to be saying whatever I wanted it to say. But more than that, although it told me things, I had somewhere lost the reality of my world and the reason for my research.

The data told me a story, but it only offered part of the story, and I could not find myself in the story. Indeed I was required to remove the “I” from my writings and replace it with the third person.

As planned, I moved into teaching from this very strong mathematical and scientific education.

Into my classroom came teacher trainers speaking of “best practice” and bringing mechanistic tools for assessing what I was doing and what I should be doing. The numbers were all there, but yet again I had a feeling that the numbers were not the whole story and I was not a part of the story.

My feeling of dissatisfaction with purely quantitative studies grew and, whereas I wanted desperately to continue with my studies, the education courses were not offering me what I sought.

I was trapped in the quanitity-quality dichotomy, still with the perception that it was a case of either or. I was as uncomfortable about this as I was about pure quantification.

I taught, and thought that I taught well, but could not pin down for myself what I did that was good and so had no means of isolating that that was in need of improvement. Maybe there was another world out there that accepted my experience and taught me how to go about researching my practice using the wholeness of my own experiences.

Brian Goodwin speaks of this tension between quantity and quality. He accepts the good of quantity: "As a strategy for exploring an aspect of reality – the quantifiable and mathematizeable – the restriction of modern science to primary qualities is perfectly reasonable. It has also turned out to be remarkably successful." (Goodwin, 1999:1) He wants, however, a method of researching other aspects of the world and proposes a science of qualities: "But is there any intrinsic reason why there should not be a scientific methodology that addresses aspects of the world that are connected with qualities? Is there any reason why qualities should not be reliable indicators of 'objective' states?" (Goodwin, 1999: 6)

I heard of a Masters course offering a different philosophy and method of research. A sign for me that it would be different was when I received a reading list, the first part of which was material from Kundera's *The Unbearable Lightness of Being*.

Shortly after starting the masters course, I attended a SMILE Conference in Manchester and listened to Professor Margaret Brown of Kings College lecture, amongst other things speaking of best practice and the search for this Holy Grail. She listed about one hundred aspects that had been studied in a few thousand international research projects and showed how, for almost all these aspects the studies were absolutely inconclusive or contradictory.

Thus the search for good practice for me moved from the classrooms of fine teachers to my own classroom. Chris Breen and John Mason within my masters programme offered a course on Re-Searching Teaching (Breen, 2001, 2002) opened a glimmer of possibility of a methodology that did not prescribe, but which rather provided tools with which one could open one's practice.

Articles from a variety of sources began to show groupings of people around the world using research methodologies that promoted the improvement of one's practice, rather than the pursuit of a global truth. Amongst others, the enactivist movement in Australasia and Canada (Reid, 1996; Begg, 2001a), the living theory advocates in Bath (Whitehead, 1999; Larter, 1988) and the discipline of noticing theorists in England (Mason, 2002; Wilson, 1996), all heavily influenced by Eastern thinking, started to add to a picture of a research paradigm that privileged the teacher's experience.

The first year of the masters passed with the completion of the coursework. At the beginning of the second year I was faced with a compulsory short course required by the School of Education on research methodology. Suddenly all those exclusionary walls of the academy that had been so effectively deconstructed during the coursework, were bulldozed back into place – seemingly twice as high and oppressive now that I had a view beyond them.

"If you don't follow a set procedure for submission – you fail." "If you allow a chink in your armour – you fail." "If you do not add to the sum of human knowledge – you fail." "If you haven't started already – you're late." "Most people start a masters and never complete it." "Choose a small question, for which data is easily obtained, write it up and submit. Get the cloth on your back, prove that you can research, then you can start asking the questions that truly matter to you."

Suddenly my study again became about gaining a piece of paper and I was no longer interested in that - I was only interested in becoming.

So I did little for a year, until my practice again forced me to think and want to know more about what I was and could be doing.

And so I started laying down the path in the walking (Varela, Thompson and Rosch, 1991), but soon became uncomfortable with the analogy. It was too filled with direction for me. So instead I started to create my space in the midst of the dichotomies to which I had been exposed - a space in which I could live and teach and have my being (Figure 1).

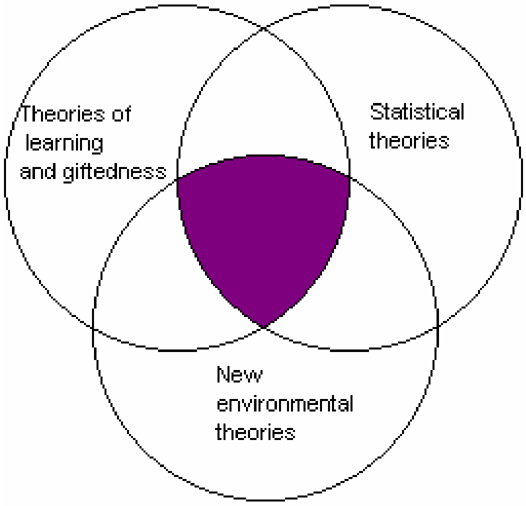


Figure 1

The theories of learning offered by Piaget and Vygotsky, the theories of giftedness from Stanley, *et al*, the wealth of statistical theories on offer (particularly those related to regression discontinuities) and the new environmental theories of, amongst others, Capra and Maturana and Varela, provided regions of extreme wealth that were open for use.

Davis and Sumara (1997: 122) in a study in an elementary school acknowledge this web of influences: "Teaching acquires its form within a complex relational web that seeks to affect the understandings and abilities of the individual members of that community."

This then became my space and from this space I could enter a variety of domains, using from each of these domains the tools and ideas that might help me to answer questions about my space. According to Smith (2001: 37), "The New Environmental Paradigm view of systems is that the 'elements' in the system cannot be viewed separate from the system itself. They are intimate partners, mutually coupled in dynamic transformation."

And in the evocative words of Bly (1990: 175):

> "To live between the opposites means that we not only recognise opposites, but rejoice that they exist. To live between we stretch out our arms and push the opposites as far apart as we can, and then live in the resonating space between them. Living in the opposites does not mean identifying with one side and then belittling the other . . . Rejoicing in the opposites means pushing the opposites apart with our imagination so as to create a space, and then enjoying the fantastic music coming from each side."

My problem was how to bring these different aspects together into a whole that would be accepted as a valid piece of research. I needed a theory that admitted disparate elements. It is here that the literature of enactivism provided a possibility.

Maturana and Varela (1987) and Varela, Thompson and Rosch (1991) took the ideas of Merleau-Ponty and extended them into a biological understanding of cognition and re-united mind and body. The mind was no longer seen to be situated in the brain and thus thought and action became part of one whole. "All doing is knowing, and all knowing is doing." (Maturana and Varela, 1987: 26)

Their theories encouraged a mind-full living of life and that "as mindfulness grows, appreciation for the components of experience grows." (Varela, Thompson and Rosch, 1991: 122)

They proposed a new word to describe this new understanding – enactivism:

> "the growing conviction that cognition is not the representation of a pregiven world by a pregiven mind but is rather the enactment of a world and a mind on the basis of a history of the variety of actions that a being in the world performs."
>
> (Varela, Thompson and Rosch, 1991: 9).

At a similar time Capra was working in the world of physics to establish a new understanding of the study of life along a systems approach. He highlighted a

> "crisis of ideas: the fact that most academics subscribe to narrow perceptions of reality which are inadequate for dealing with the major problems of our time. These problems are systemic problems which means they are closely interconnected and interdependent."
>
> (Capra, 1982: 6)

The stage was set for a "post-modern anti-grand-narrative critique." (Begg, 2001b: 13).

Davis (1996) and Davis, Sumara and Luce-Kapler (2000) began to engage with these trends by incorporating them into a theory of the teaching of mathematics.

It is within this context that I situate my work, overlapping parts of the modernist tradition with parts of the post-modern. I tell my story as I have experienced it, in a way that is useful to me. Research in education, for me, has ceased to be about adding to that mythical body of human knowledge. It is far more about my growth as a researcher, as a teacher and as a person. My story, as I hope that you will find, although not in a standard format, is also not atheoretical – it tries to use the variety of theories available to theorise about what I am doing. This is what is useful to me.

COLLECTION 3
OF MY THOUGHTS AND ACTIONS:
WHERE A SOUTH AFRICAN AD MATHEMATICS PROGRAMME BEGINS

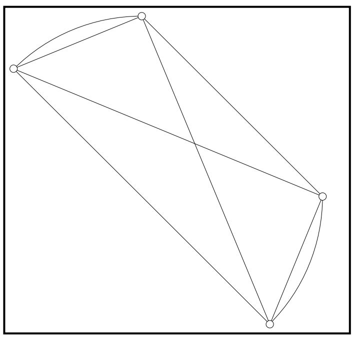
A section in which I consider the theories of giftedness that currently exist and in which I describe the Ad mathematics programmes through which I have taught and learned. From the confines of what I know and have done, I stare out, not sure whether there is more that I could be doing, but with a feeling that there are unexplored worlds beyond those with which I am familiar.

"Alvin stared out towards the limits of his world. There was nothing beyond them – nothing at all except the aching emptiness of the desert in which a man would soon go mad. Then why did that emptiness call to him, as it called to no one else that he had ever met?"

(Arthur C. Clarke in *The City and the Stars, 1956*: 28)

"The only concept of an educated society which is not merely utopian is a society of which the members should all have reached their full and proper development along the lines of their own potential excellence. Such a society would certainly not be an egalitarian one. [People] are not born equal, and cannot be trained to reach equality; like other living things they are immensely and marvellously diverse, which is a fact to rejoice in, not to lament or seek to obscure. In any society the men and women with the best minds are, and always have been, the cream . . . and it is they who in all ages have given the tone to the society in which they have lived. I think we must admit that to train these minds is the most vital and important object of any system of education. It is by no means the only object; but if it should fail in this, it fails altogether."

(Aubrey de Selincourt, quoted from *The Schoolmaster* in de Jager, 1964:1)



Much work on the gifted has emerged from the United States, where programmes of acceleration (as opposed to enrichment) have been followed. The work at the Johns Hopkins University (see for example Stanley, Keating and Fox (1974) and Stanley, George and Solano (1977)) provides numerous case studies of pupils who were accelerated through the school educational system into the universities at very young ages. The work that they completed through their two projects, Study of Mathematically Precocious Youth (SMPY) and the Intellectually Gifted Child Study Group (IGCSG), gives a number of insights into the selection of pupils for such programmes and the effect that these programmes had on their intellectual development.

There is equivalent evidence for the success of gifted programmes that follow an enrichment methodology, i.e. teaching material not usually contained in the syllabus or any future syllabus, material that broadens concepts usually covered in syllabi at that level of the child's development (Wavrik, 1980). These programmes often emphasise and develop problem-solving skills.

Robinson and Stanley (1989) evaluated a successful programme in Arkansas that followed a combined enrichment and acceleration methodology.

Canadian projects, particularly the University of Toronto (Matthews, 1997; Kalchman and Case, 1999), have continued much of this research that was based in the United States. Their emphasis has moved away from the original idea of categorical giftedness (i.e. a child being highly talented in all areas) to a domain-specific concept of giftedness (i.e. a child being talented in particular knowledge and emotional areas). The research of Kalchman and Case has also emphasised the importance of within-class variation even within a top of the spectrum grouping. The need for teachers to acknowledge this and to allow children to progress at their own pace is of relevance to the South African context, where within-class social variation is even greater than that in the Canadian context.

Much of this work that has evaluated gifted programmes was situated in primary schools because of the large number of other influences related to maturation that might influence the data in high schools.

In the South African context there is a dearth of research aimed at the mathematically talented pupil. The little that is available is dated and therefore reflects information only on the White segment of the population. Mostly it is aimed at understanding what should be taught in an Ad programme with little on the effects of this on those who have been exposed to the programmes (Breen, 1980, 1982; Gouws, 1977 and de Jager, 1964).

When I began my teaching career, I taught at a state school with a very well developed ethos of Ad mathematics that had been built over a number of decades. These programmes existed, in various forms, at a large number of private schools across South Africa, but were only found at a very limited number of state schools. When I moved to another school three years ago, part of my job was to institute an Ad mathematics programme.

I wish to give an overview of what constitutes these programmes, before going into a motivation of the need for an increased number of these opportunities locally.

At the end of a first year in high school (grade 8 – 13/14 year olds) a class of the most talented mathematics pupils is selected to form an Ad mathematics class. This selection is based primarily on marks and ability to engage with mathematical problems. This class is, ideally, taught by a single teacher for the remainder of their mathematics schooling. It is up to the individual teacher (and in some cases class) to design a syllabus.

The work that they cover is of a three-fold nature:

- ♦ Ordinary work: Work usually covered by that grade during the year.
- ♦ Accelerated learning: Work taken from school syllabi of more advanced years.
- ♦ Enrichment learning: Work that would either not normally be covered at school, or work that extends syllabi topics to a level that would not normally be covered at school level.

All three types of work contain a strong component of problem-solving, lateral thinking and deeper thinking. The slightly odd term describing the class – "Ad" – is used to describe the two types of work that are different to ordinary, i.e. advanced (accelerated) and additional (enrichment). It is often an expectation of the programme that pupils in the programme should be actively involved in mathematics Olympiads and competitions.

The class is run completely separately to the ordinary stream with different tests, projects and examinations. The assessments are naturally of a greater difficulty and consequently the pupils expect their marks to be lower than in an ordinary grade class.

Once these children get to matric (final school year at 17/18 years of age) they revert to the ordinary grade of tests and examinations to prepare them for their school-leaving certificate examinations. It is an expectation that their marks and insight when they revert to ordinary level working should be of a higher level than would have been expected if they had continued without an Ad programme. Part of the role of this dissertation is to assess whether there is any evidence to back this assertion.

Any child who wishes to remove him or herself from the programme can do so at any stage and will slot back into the ordinary grade classes.

For those within the class who are very competent in mathematics, there is the opportunity to write an extra mathematics subject in their school-leaving examinations. This syllabus contains material from the traditional first year mathematics courses at universities. In this dissertation I have not covered aspects of this course. It is an important component that requires further research. There is anecdotal evidence from university sources for negative effects of this on those who study mathematics at a tertiary level.

George (1976) identifies 5 essential components of a successful accelerated programme:

- Effective identification of pupils for the programme.

- Selection of a dynamic teacher.
- Compatible learning styles.
- Development of positive work habits.
- Self-motivated pupils who are not in the programme under duress.

In the design of the programme I offer, I have leant on some of the developmental work of Jean Piaget (Piaget, 1941, Piaget, Inhelder and Szeminska, 1960) and Lev Semenovich Vygotsky (1962, 1994) which provides a basic understanding of the need to acknowledge levels of development within a child. For teaching to be effective, the children must be challenged at their level. Too high a level will lead to negative self-image, while too low a level will lead to boredom. Vygotsky suggests a zone of proximal development in which a pupil will learn.

Although much of Piaget's work has been superseded by Vygotsky and others, Piaget offered one of the first systematic studies of the linear development of intellect in young children. He offered it as "an analytic study of the beginning of measurement . . . to those who are concerned to foster an educational approach which is securely based on a knowledge of the laws of mental development." (Piaget, Inhelder and Szeminska, 1960: vii)

Piaget (1941) draws interesting links between the evolution of perception in a child and the historical development of mathematics. Because of this process of intellectual development, Piaget suggests that although it is possible to accelerate certain concepts, others will merely be learned without understanding if they are taught at the wrong level. True understanding will not be possible if the child has not progressed to the correct stage. An obvious example is the teaching of formal axiomatic geometry.

Linked to this, Piaget also emphasises the difference between the child and the adult brain, that this difference is not quantitative, but qualitative. Expecting a child to simply accelerate their learning, because they know quantitatively less than the adult is not viable. The level of their development may preclude the understanding of some concepts.

Vygotsky, like Piaget, also distinguishes between the adult and the child and contends that it is impossible for an adult to pass on to the child their way of thinking.

The concept of resonance emerges frequently in Vygotsky's work - that a new concept or structure will gradually spread into the older concepts. As one learns something new, it will slowly transform previous concepts. This has enormous implications for the teaching of an Ad mathematics programme, where often a particular topic of mathematics is visited only once, in extreme depth, and not revisited. Vygotsky's work suggests that returning to concepts repeatedly at various levels will be beneficial and allow "higher order concepts [to] transform the meaning of the lower" (Vygotsky, 1962, ix). This also suggests the need to make overt in the classroom the new and powerful meanings that algebra gives to arithmetic. "[T]he adolescent who has mastered algebraic concepts has gained a vantage point from which he sees arithmetical concepts in a broader perspective." (Vygotsky, 1962: 115).

For myself, the aims of an Ad mathematics programme that I offer are reasonably diverse:

- to provide an environment in which mathematicians and those who enjoy the subject can develop.
- to provide an environment in which future academics (those who will continue academic work, particularly as teachers, through their adult lives) can develop.
- to produce thinkers with the tools to deeply analyse the world in which they live.
- to allow the talented pupils a means of developing themselves to the limit of their ability.
- to promote the enjoyment of mathematics.
- to challenge and refresh myself as teacher.

Currently the South African education system is undergoing marked structural changes towards a system of outcomes-based education. These changes are having a substantial effect on both what is being taught in the classroom and how it is being taught. The documents for the grade 8 and 9 learning outcomes (formerly syllabi) are currently being implemented. The equivalent documents for the grade 10 to 12 levels are in rudimentary form and are under discussion and field testing.

The Curriculum 2005 documents (the guidelines for national implementation of the system of outcomes-based education) require teachers to allow all pupils the chance to progress at their own pace, until they achieve mastery of relevant and stated knowledge, skills, values and attitudes. "The Descriptors are provided to help educators and parents . . . so that appropriate interventions can be set up to help with learner development or to provide for the rapid development of gifted or highly proficient learners" (Western Cape Education Department, 2000: 3).

It is my feeling that South Africa requires of its mathematics education system two types of people:

- firstly, those who know sufficient mathematics to operate effectively in a highly numerate and technologically advanced society, that requires of its members good, logical skills, mathematical literacy and problem-solving ability
- secondly, those who know mathematics at a higher level and will be able to pursue tertiary courses in the subjects that lead to the engineering, technicist, computer, etc. professions.

It is my impression that at present a system of Ad mathematics that allows the development of this latter group is desperately needed for a number of reasons:

- The country's shortage of qualified professionals who can take up positions, particularly in the various engineering and educational fields. This shortage is being fuelled by the draining of a large percentage of qualified young adults to other countries, due to the perceived political and economic situation in South Africa.
- The disproportional shortage of highly qualified students from the historically disadvantaged groups. Only ten years ago, of every 10 000 black pupils who enrolled for the matric examination, only 1 passed with exemption (a grade that would allow entry to a university) and higher grade mathematics and science.

This population group at the time composed 76% of the total pupil enrolment (Department of National Education, 1991).

- The new education system requires that all pupils, including mathematically talented ones, be afforded the means to achieve their full potential.

  > "[W]e will have to consider the long established and scientifically observed fact that where the environment fails to present appropriate problems, does not come up with new requirements and does not stimulate and create development of the intellect with the help of new goals, the adolescents thinking does not develop according to all the available potential, and it does not reach its higher forms, or only achieves them at an exceptionally late stage." (Vygotsky, 1994: 214)

- Through the teaching of an ad mathematics programme, teachers will again be required to take on the role of learners, revisiting material that they last attempted at university, and possibly be inspired to study new material. It will again be possible for them to become students of their subject (a practice now long neglected in the profession), not just people who transfer certain knowledge and procedures.

It is heartening to see that new programmes are currently being set up in numerous other schools by some excellent mathematics teachers. There is therefore now a pressing need for research to inform and guide such programmes.

For my dissertation I have used data from a number of different Ad programmes:

- An Ad mathematics class taught by myself at my second school from 2000 (grade 10) to 2002 (grade 12 / matric).
- My current Ad mathematics class taught during 2001 (grade 9) and 2002 (grade 10).

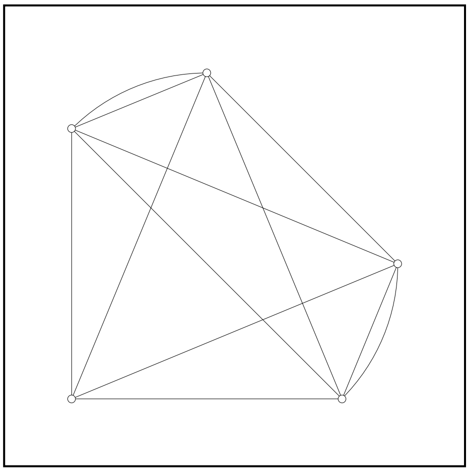
This data comprises a number of forms – end of year mathematics marks, interviews, video material from my classroom, my written reflections, the written reflections of pupils in my classes and questionnaire responses. "These artefacts can be lumped together as “data”, but at the same time all of them record acts of interpretation, or a researcher learning in co-emergence with a research situation. It can be said that there is no data, only interpretations and interpretations of interpretations.” (Reid, 1996: 205). A reflection of mine after a lesson could lead to a questionnaire to the pupils, whose responses therein would lead me back to re-viewing video material from my classroom and so the loop continues.

COLLECTION 4
OF MY THOUGHTS AND ACTIONS:
RE-COGNISING THE PUPILS' SPACE IN AN AD MATHEMATICS PROGRAMME

A section in which I begin to re-cognise the experience of those pupils who have experienced an Ad mathematics programme. I attempt a combination of quantitative and qualitative data to open up the story as fully as possible.

"On rare and unforeseeable occasions, the Jester would turn the city upside-down by some prank which might be no more than an elaborate practical joke, or which might be a calculated assault on some currently cherished belief or way of life.
Alvin knew with a certainty that passed all logic, that the welfare of the race demanded the mingling of these two cultures [of the cities of Diaspar and Lys]."
(Arthur C. Clarke in *The City and the Stars, 1956*: 51, 190)

"Qualitative evaluation research, and the marriage of it to the more readily-accepted quantitative procedures, is likely to have a dramatic effect on the quality of gifted program evaluation."
(Archambault, 1984: 23)



SUB-COLLECTION 4a

OF MY THOUGHTS AND ACTIONS:

ON THE SURFACE

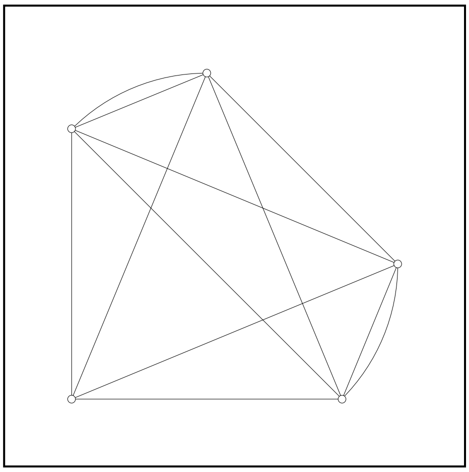
A sub-section in which I begin to point statistical tools at my practice. A regression discontinuity analysis is performed to assess whether there is any noticeable difference between the results of an Ad mathematics group and a non-Ad mathematics group.

"Jeserac [the teacher] could find how numbers behaved, but he could not explain why . . . But he was fascinated by the way in which the numbers he was studying were scattered, apparently according to no laws, across the spectrum of the integers. He knew the laws of distribution that had already been discovered, but always hoped to discover more."

(Arthur C. Clarke in *The City and the Stars, 1956*: 49)

"Descartes' method of thought and his view of nature have influenced all branches of modern science and can still be very useful today. But they will be useful only if their limitations are recognised. The acceptance of the Cartesian view as absolute truth and of Descartes' method as the only valid way to knowledge has played an important role in bringing about our current cultural imbalance."

(Fritjof Capra in *The Turning Point, 1982*: 43)



When assessing the teaching and learning that occurs in a class of mathematically talented pupils, one is faced with a difficult job. Statistically it is difficult to compare the performance of these pupils with the others, because they are from different sections of the normal distribution. By the definition of their mathematical talent, the results of those in an Ad mathematics programme should show a statistically significant difference with those who are not in the programme.

The fact that those in an Ad mathematics programme are involved in testing and assessment of a different level to those in an ordinary mathematics programme further complicates assessing the effectiveness of the programme. As Archambault (1984: 12) acknowledges, "The evaluation of programs for the gifted and talented has become an increasingly frustrating challenge."

Stanley and Robinson (1986) and Robinson and Stanley (1989) provide a means for considering the effects of a programme on different ability groupings using a statistical technique termed regression discontinuity. Their methodology involves contrasting test scores on common tests of both the gifted programme pupils and those not involved in the programme prior to the programme and after the programme. Linear regression lines are then plotted for the two populations. The discontinuity between the two lines at the 'cutting score' (the percentage score on the pre-test for selection into the programme) will then give an indication of the effectiveness of the programme (Figure 2).

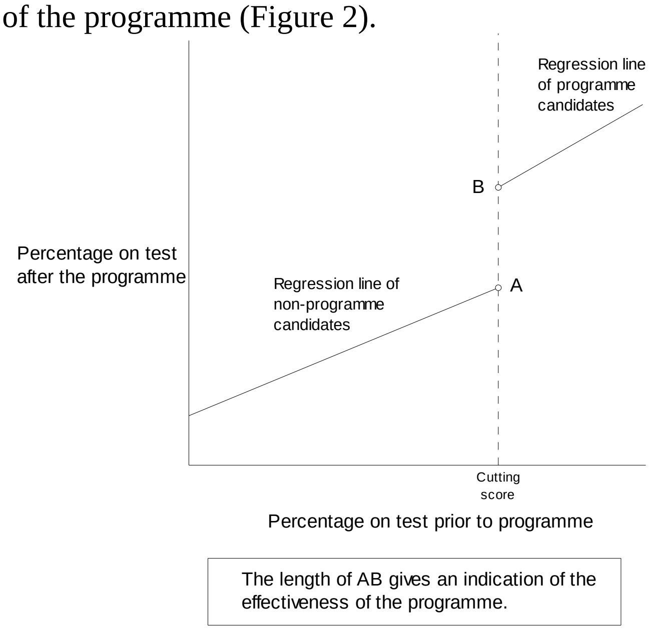


Figure 2

Here I take the results of the grade 12 class that wrote their school leaving examinations at the end of 2002. The one part of the class is 25 pupils who took part in an Ad mathematics programme that I taught for 3 years. Two of them are excluded from the analysis – in the one case the pupil was not at the school in grade 8; in the other the pupil's matric results were not yet available. It is a class of both boys and girls of mixed races at a state high school in Cape Town. The other part of the class is 25 pupils who wrote the same higher grade matric examinations at the end of 2002, but who were not members of the Ad mathematics programme for the full duration of the course.

Both groups were examined under the same conditions in grade 8, and this provides a single pre-test score for the whole sample.

Firstly I have simply plotted a scatter graph of their grade 8 versus their grade 12 results and drawn two separate regression lines based on the least-squares method.

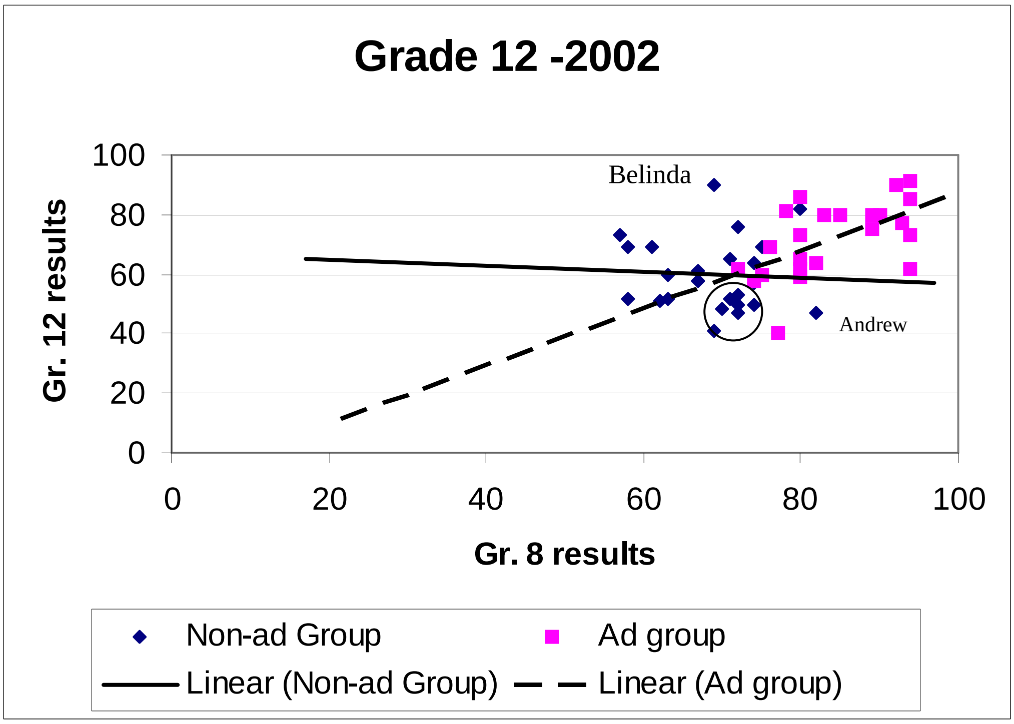


Figure 3

My layering of quantitative material into qualitative material under the framework of a Discipline of Noticing after Mason (2002) is dealt with in more detail in a later chapter.

I acknowledge my assumption that the regression of marks will be linear, but feel that this it is acceptable based on the work of Stanley and Robinson (1986). Because of the scatter of the data, the R^2 values of neither graph are high. Both graphs however, do tell stories that are worth interrogating.

Thus I wish to attempt to allow this graph to open up understandings of what interpretations could be drawn from this situation.

Interpretation 1:

The Ad mathematics programme has substantially improved the results of those who experienced the programme.

Evidence 1:

The two regression lines are of a significantly different nature. The gradient of the line of the Ad mathematics group is substantially greater than that of the non-Ad group, meaning that those at the top end of the Ad mathematics class have attained higher marks than would have been expected. If the regression line of the non-Ad group is accepted as what would happen without an Ad mathematics programme, 21 of the 23 in the Ad mathematics group are attaining marks above this expected level.

Discussion and further research 1:

Is there evidence in a larger sample of pupils that would validate the positive effect of an Ad mathematics programme on grade 12 results?

Interpretation 2:

The Ad mathematics programme would be detrimental to the marks scored at the end of grade 12 to those in the lower end of the Ad mathematics class.

Evidence 2:

The two regression lines intersect at an x value of approximately 75%. Any pupils with less than this result in grade 8 could be expected to be negatively influenced by the Ad mathematics programme.

Discussion and further research 2:

Through the written reflections of those at the lower end of Ad mathematics classes, is there evidence that they feel they are being negatively influenced?

Kalchman and Case (1999: 326) give strong evidence for the diversity of gifted classes and that the assumed "homogeneity of ability [in these classes] should be reconsidered." My evidence shows a possibility that those who were just above the cutting score for inclusion in the Ad mathematics programme have gained little benefit, possibly due to an assumption in the programme that all pupils in the class could work and grasp concepts at an equally rapid rate. Further programmes should carefully track these pupils to find out how they learn best and should allow them the space to move at their own pace.

Interpretation 3:

Although the Ad mathematics programme has a positive influence on those in the Ad class, it has had a negative effect on those in the rest of the school.

Evidence 3:

The slightly negative gradient of the regression line of the non-Ad group is surprising and not present in any other data studied of the same school in previous years when there had been no Ad group. The grouping on the graph that has been circled is a cause for concern. They were relatively high achievers in grade 8, but have dropped off significantly by grade 12.

Discussion and further research 3:

Could this be due to negative perceptions of themselves when they have interacted with the Ad mathematics grouping in other subject classes?

Interpretation 4:

This interpretation attempts to get away from the whole picture, down to a single story. Andrew's interpretation (throughout my dissertation I have changed the names of pupils to protect their anonymity) of the above graph in assessing the Ad mathematics programme could well be that the Ad programme has significantly detrimental effects on those who leave the programme.

Evidence 4:

Andrew was a high achiever in grade 8. As a result he entered the Ad mathematics programme. He moved back to higher grade during the course of his grade 11 year, because he felt that he was no longer coping with the Ad mathematics environment. He continued to struggle with mathematics, and only managed to obtain a result marginally above a pass by the end of grade 12.

Discussion and further research 4:

Were there other issues in Andrew's life that resulted in this decline that had nothing to do with Ad mathematics, or were his poor grade 12 results somehow linked to his participation in the Ad mathematics programme?

Interpretation 5:

Belinda's interpretation may be that an Ad mathematics programme has no effect on a pupil. Good marks can be obtained by simply putting in a lot of work during your final matric course, cramming in the techniques prior to writing your tests and examinations.

Evidence 5:

Belinda worked adequately in grade 8, but she seldom pushed herself, attaining a result just below 70%. The consequence was that she was not invited to be a part of the Ad mathematics class. Late in her schooling she suddenly started to work hard to prepare herself for her final matric examinations and scored excellent marks on these papers.

Discussion and further research 5:

Are there non-mark-related benefits to Ad mathematics that Belinda has missed out on (particularly given the concept of a ceiling to the marks attained by high achievers as discussed by Archambault (1984))? Archambault further discusses the lack of tests that effectively evaluate higher order thinking skills that may be gained in an Ad mathematics programme.

Interpretation 6:

An Ad mathematics programme has no effect on the marks of pupils in grade 12.

Evidence 6:

There is no evidence to show that the above difference in regression lines is not a normal occurrence, i.e. that a normal top set, in a school where no Ad mathematics programme occurs, usually shows a significantly different regression to the rest of the higher grade pupils.

Discussion and further research 6:

Do regression lines of different sets in schools in which there is no Ad mathematics programme show any significant difference to those of the remaining pupils?

All of these stories (and countless others) are valid interpretations of the graph. For each interpretation, counter-arguments and "accounts for" can be offered, but no absolute truth can be obtained. Bearing all the interpretations in mind, while working with further evidence through the course of this dissertation, will allow me to more fully understand the whole situation of an Ad mathematics programme.

I then took the exact same data and performed a regression discontinuity analysis in the manner of Robinson and Stanley (1989). I take the data from both groupings and fitted a regression line of the following form to this data:

$$y = ax + b + cz$$

where: y is the score obtained in the post-test (grade 12 results)

x is the score obtained in the pre-test (grade 8 results)

a is the coefficient of regression (gradient)

b is a constant

z is a dummy variable (1 in the case of those who completed the Ad mathematics programme and 0 in the case of those who did not.)

c is the mean effect that the programme has had.

An analysis of the data using this method yields a regression line of:

$$y = 0{,}5x + 26{,}9 + 4{,}8z$$

Indicating that the programme in that year has had an average effect of raising the marks of those who have been a part of the programme by 4,8% from what they would have been expected to achieve given no programme. The R^2 value is, however, only 0,26.

A simplified schematic representation of this would be:

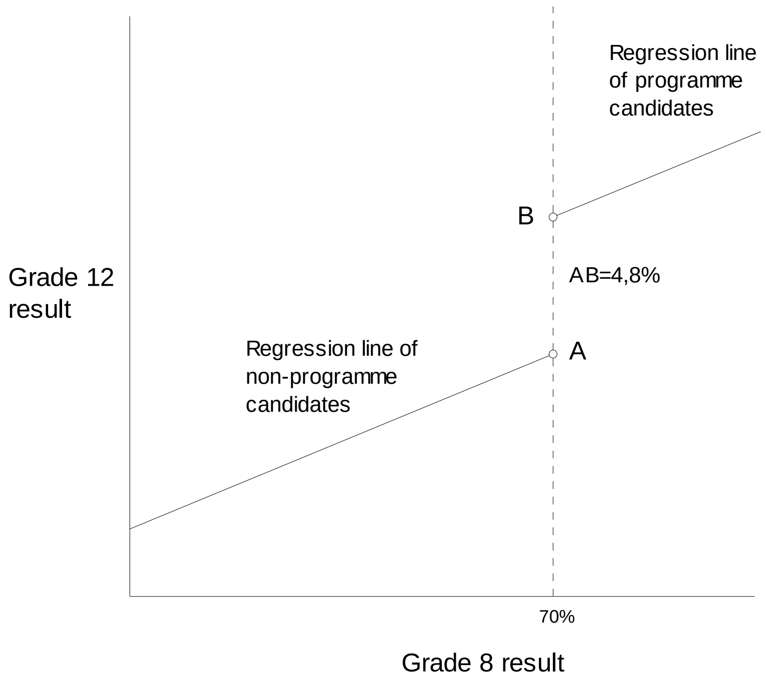


Figure 4

Trochim (2002) notes that "the discontinuity could be a change in level vertically or a change in slope or both." The low value of R^2 is, I feel a combination of two factors:

1. The scatter of the data.
2. The indication in Figure 3 that the effect of the programme has not been to shift the whole regression line of the Ad mathematics group, but rather to steepen its gradient, thus only affecting a portion of the class.

This method of regression discontinuity I feel holds strong potential for use in schools, not only to evaluate Ad mathematics programmes, but also to evaluate the relative success of classes where streaming is used. I need to perform further research on future programmes of Ad mathematics and on the success rates in other schools where such programmes have been implemented.

SUB-COLLECTION 4b
OF MY THOUGHTS AND ACTIONS:
THEN GOING DEEPER

A sub-section in which I take some first steps out of the number-based research methodology with which I am comfortable. I consider here the threads of certain moments recognised by my pupils and I attempt to re-cognise these in the light of the statistical results of the previous sub-section. The data is more difficult to work with, but holds the promise of something deeper.

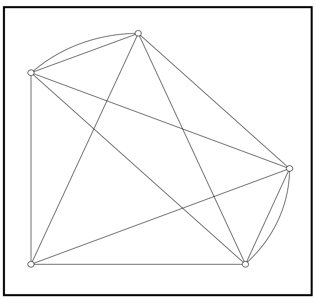
"I am trying to find a way out the city," Alvin said bluntly. There must be one, and I think you could help me find it." Khedron [the jester] was content with the order of things as it was. True, he might upset that order from time to time – but only by a little – he was a critic, not a revolutionary. Yet he still possessed that spark of curiosity that was once Man's greatest gift. He was still prepared to take a risk.

Sympathy, for one whose loneliness must be even greater than his own; an ennui produced by ages of repetition; and an impish sense of fun – these were the discordant factors which prompted Khedron to act. "I may be able to help you, or I may not. I don't wish to raise any false hopes."

(Arthur C. Clarke in *The City and the Stars, 1956*: 57 - 58)

As Poincare would have said, there are an infinite number of facts about the motorcycle, and the right ones don't just dance up and introduce themselves. The right facts, the ones we really need, are not only passive, they are damned elusive, and we're not going to just sit back and 'observe' them. We're going to have to be in there looking *for them or we're going to be here a long time. Forever. As Poincare pointed out, there* must *be a subliminal choice of what facts we observe . . . The difference between a good mechanic and a bad one . . . is precisely this ability to* select *the good facts from the bad ones on the basis of quality. He has to* care*!*

(Robert Pirsig in *Zen and the Art of Motorcycle Maintenance, 1999*: 285)



John Mason of the Open University (1995, 1998, 2002) is pioneering work in a research method which tries to establish the teacher as a researcher, tries to equip the practitioner with tools to generate meaningful research in his or her own context.

The Discipline of Noticing involves a number of procedures:

1. During practice a researcher will mark certain events as being meaningful, as being worthy of notice.
2. He will later return to these and attempt to record a brief-but-vivid ‘account-of’ the occurrence. In this recording, an attempt is made to give no justification or explanation of the occurrence. In this way, the researcher tries to train himself to observe situations without judging them and to attempt to record, as impartially as possible, these occurrences for future reflection.
3. Once a series of accounts is available, strands or trends are grouped.
4. The researcher then interrogates her past practice or the experience of others, offering these accounts to see whether they have resonance with others or with her own past experience. One technique used by the researcher is to construct a task exercise – a task that is offered to another person in an attempt to give that person entry into a situation to probe for resonance.
5. Multiple interpretations of the occurrence can then be considered. This allows the researcher time to develop, to gather from others or to find in literature, gambits (ways of reacting to similar situations) for the future.

Lerman (1995) and Ernest (1999) have offered criticisms of the discipline of noticing, amongst others the implicit nature of the knowledge and the lack of validation of moments attributed to the partiality of the researcher. In response to certain of these issues I have used certain aspects of the Discipline of Noticing and sought validation through including a variety of techniques in my research.

In my seeking validation I have extended Mason’s concept of an ‘account-of’ to include quantitative stories.

In sub-collection 4a I have utilised the graphs themselves as ‘accounts-of’ a situation as envisaged by Mason (2002). They describe briefly and vividly a story without ‘accounting for’ the situation. I was then in a position to make multiple interpretations of these accounts in pursuing my understanding thereof. Reid notes that these multiple interpretations may be attained in a number of ways. “One is through multiple revisitations of data which bring a researcher to a situation with new theories and aims which represent the current structure of an ever changing being.” (Reid, 1996: 209)

I now focus my inquiry on those 25 pupils who have just completed the grade 10 course that I have offered in Ad mathematics. My study is based on two sources of data:

- My own reflections and notes on teaching the class.
- The reflections of the pupils from written work that they submitted to me to be included in their portfolios (the notion of how a portfolio system was implemented is explained later).

I used this data as my source of “accounts-of” my practice that I then interrogated, searching for common threads. Four of these threads are what I report here.

Preceding each set of reflections is a brief task exercise for the reader to attempt prior to considering the story that I have experienced. These will hopefully engage the reader more fully in that experience. The analysis is then split into three parts – pupil reflections, my reflections and a theoretical discussion of the two.

The question of the validity of such data needs to be addressed. Hardy, Hanley and Wilson (1995: 2): “How much does this matter? I want to suggest, that from the viewpoint of a reflective practitioner, the issue of validity is much more one of whether the retold anecdotes are recognisable by other practitioners, resonate, are enterable, are discussible than of whether they are ‘true’ or not.” Validation lies in the responses of the reader and in resonance with previous experience that the experience holds for the teller.

FIRST SET OF REFLECTIONS:
COVERING TOPICS OTHER THAN MATHEMATICS

TASK EXERCISE 1:

On a sheet of paper, without any reflection, jot down the first five things that spring to mind that you remember learning at school.

Now, after some reflection, jot down what you consider to be the five most important things that you learned at school.

PUPIL REFLECTIONS

Mary:

"Why do we have to do maths?" . . . I just wish people would answer the question right . . . Because in maths you learn. But the problem is that the average teenager does not get excited about learning. I am very glad that I took Ad maths, because I've learnt so much. I've learnt how to enjoy maths for being more than maths and I am really glad that I don't not like maths anymore.

Julie:

I think [that in this class] we all learn a lot and not only about maths but also each other and many other things.

Maria:

I have learnt many things in Ad maths about both mathematics and life that I wouldn't even have touched in normal maths.

MY REFLECTIONS

1. I am excited about the class that is coming in the door – I have been thinking through the whole summer holidays about what I could achieve with them in an ideal situation. It is my first experience with beginning an Ad mathematics group at this school. They come in and we greet each other. We all sit down and there is quiet – they are all looking at me. I say, "And so there we were . . ." When silence follows, there is a bit of laughter. The silence becomes awkward, with some pupils fidgeting and looking down. Maria volunteers, "So how was your holiday, sir?" "Good," I reply. Silence again. The conversation continues sporadically for the rest of the period.

The period the next day begins. I put my hands out towards the class and make as if to say something. "And so there we were . . ." a couple volunteer. Laughter follows and then again silence. "Aren't we going to do anything," Bianca asks. "What would you like to do." "Talk about things." "Alright – what." And the rest of the period wanders through a variety of matters.

The third lesson begins. All now volunteer, “And so there we were . . .” “When are we going to do some maths?” Mark asks.

2. Just before they leave school, I ask a group of matric pupils who have been in my class for four years, to write down the five most important things that they think they have learned at school. They then all volunteer their lists to the class and we make a complete list on the board. Not a single item on the list has anything to do with mathematics or any one of their other subjects.

SOME THOUGHTS TO COMBINE THE TWO

To be an effective teacher it is important for me to consider what I am teaching, why I am teaching it and how I am teaching it. The Ad mathematics programme opens space, away from the constraints of a syllabus to pursue issues that the pupils or I consider to be important. This notion of completing work other than mathematics does lead to some conflict with pupils and parents who consider the work to be covered in a narrow syllabus format.

SECOND SET OF REFLECTIONS:
FRUSTRATION WITH DIFFICULTY

TASK EXERCISE 2:

Prove that the product of two consecutive numbers is divisible by 2.

Prove that $n^2 + 3n + 2$ is divisible by 2 if n is a natural number.

Assess the difficulty that you experienced proving each of these statements and the enjoyment that you derived from each.

PUPIL REFLECTIONS

Jaclyn:

I have enjoyed almost all sections of maths that I have been taught. Sometimes I find a section of maths difficult and have to spend a long time trying to understand it, but I don't think that something being difficult is any reason not to at least enjoy trying to work it out.

Jaclyn:

Sometimes I want to run away from maths and pretend it never existed. I wonder why it was ever invented. However, there are so many valid reasons answering my question, that I know it's caused by my feelings of disappointment . . . I think that my enjoyment of maths depends on whether I understand it or not, which is a direct contradiction to a statement in [one of my previous reflections]. I can enjoy anything I do as long as I can do it.

Jaclyn:

I feel much more confident about myself, my abilities. Instead of wanting to run away from maths now I want to embrace it.

MY REFLECTIONS

I am speaking to Joan about her mathematics choice for the following year. She wants to drop Ad mathematics and return to higher grade. I ask, "Why?" She responds that her marks have dropped too significantly. I point out the she obtained over 70% for Ad mathematics, which is only 10% below her previous year's marks. She says that it is because she does not want to have to work as hard as she does to obtain those marks. I feel myself becoming angry, not at her, but at a sense of someone backing off from a challenge.

SOME THOUGHTS TO COMBINE THE TWO

The changes in Jaclyn's thinking over the space of a year record for me the sense of personal fulfilment that I would like all my pupils to achieve. Her first reflection was written soon after she began the Ad mathematics programme, when the idealism and enthusiasm of the class was still high. The second reflection was written about mid-year just after a test on a substantial chunk of difficult work on factorisation. This was the first time that I had truly pushed the class in the level of difficulty of the mathematics that I presented. Her third reflection was written just prior to the end of the year. She has experienced the full cycle of overcoming difficulties in mathematics and has found added enjoyment based on the difficulty of the work.

In the task exercise I was hoping that you might experience a similar “a-ha” when realising that the two questions are no different, but that the thinking and eventual conquest of the second bears more enjoyment than that of the first, because of the level of difficulty involved. Many people get the first question, but really struggle with the second. It is a valuable lifeskill to be able to be able to manage this frustration.

When presenting difficult material, one is always faced with the need for a balance between challenge and the frustration of something too difficult. That is to keep the task within the zone of proximal development as written about by Vygotsky (1962). I need to be aware of those in the class who I am forcing out of their zone into an uncomfortable space.

How do I ensure that all pupils experience the full range of emotion that Jaclyn reflects? How do I prevent them opting out of a difficult situation such as in the case of Joan?

THIRD SET OF REFLECTIONS:
PORTFOLIOS

TASK EXERCISE 3:

Recall a piece of work that you have completed today (or yesterday) and consider the skills you utilised in completing that work.

How many of those skills are
similar to those skills you developed when

- you prepared and wrote a test or examination at school?
- you completed some project work or self-study at school?

PUPIL REFLECTIONS

Julie:

The portfolios are good because they can improve your marks and also they give you freedom to explore maths as much as you want and find things out that you would not have looked at otherwise.

Jaclyn:

I learned more and more as each [project] was handed back to me. I was delighted when I received my first A . . . I then knew that I could achieve things – and it inspired me to work harder.

Jaclyn:

The portfolio system teaches me much more than teachers ever could. Because I have to find out information myself, that information becomes a lot more personal and therefore easier to remember and to apply to everyday occurrences . . . this freedom means that portfolio pieces are a lot more personal and special to me than tests are.

Paul:

[The portfolio system] tends to open your mind to what maths is really about, without you even knowing it.

Julie:

If I didn't get the mark I had hoped for I was upset but I soon realised that I had learnt more than any mark could ever be worth.

Jaclyn:

Some people can do very well in tests and enjoy doing them, but I don't think I'm one of those people. I prefer to have my own space and time. That's one of the benefits of the portfolio system.

MY REFLECTIONS

One of my goals in the teaching of this Ad mathematics class was to attempt to get them to work on their own and to navigate their own way through mathematics. At the beginning of their grade 9 year (2001) I set them a job of developing a portfolio of 20 pieces of work which, by the end of the year, would demonstrate the depth and breadth of their mathematical understandings. This portfolio would count 50% of their final mark. What constituted an acceptable piece was left fairly broad, but could include reflections, solved problems, self-studies, reports on a book that they had read that had changed their thinking, sets of exercises on work that they wanted to revise, old mathematics competitions, etc.

Submission of a piece that was mathematically correct guaranteed an initial result of 60%. They could then gain higher marks by explaining their thinking (+10%), extending the work (+10%), making it original (+10%) and finally presenting the work excellently (+10%). At any point they could, after having added to the work, re-submit it to me for re-evaluation.

I remember many anxious moments on their part, trying to find ideas for topics, trying to complete pieces after having left them too late, trying to make their work original. I remember anxious moments on my part as I tried to wade through hundreds of submissions (many just before the end of the year) and give adequate responses.

I recall my frustration with a couple of pupils who did not submit any pieces (or very few). I recall frustration with a boy handing in a piece on the history of mathematics that was plagiarised. I recall frustration at finding out that some of them were copying multiple choice answers to mathematics competitions.

I also recall being ecstatic about some incredibly fine pieces of work that showed a level of mathematical understanding that I could not have hoped to teach in an ordinary classroom.

I was faced with whether to continue with the system in their next year. Was it worth the effort? Would they wish to continue?

And so at the beginning of their grade 10 year (2002) I asked them for a discussion of the portfolio system. I heard about their frustrations and their victories. They heard about my frustrations and joys. We tried to logically construct the advantages and disadvantages of the system. And then I put it to the vote. Four of the class were against it and the rest wanted to continue, with certain changes. The new portfolio system counted slightly less marks (a compromise to the education system which requires examinations in the senior years (grades 10 – 12) to count more), would only contain 8 pieces (their compromise between those who did and did not want the system) and those pieces would be of a slightly greater depth (my requirement).

I also changed my marking style, because I had become increasingly uncomfortable with the difficulty and validity of assigning a percentage to, for example, a deeply reflective piece of work and a set of exercises. Thus in the second year of the system, only one mark was available – 100%. To attain this I simply had to accept the piece as a complete piece of work. Each piece was given a letter of response in which I might raise some questions that I felt could lead to learning. They could then resubmit as often as they liked to upgrade their piece to being acceptable.

Again a large number of excellent pieces were submitted of an increasingly diverse nature. My sense, however was one of some frustration amongst class members with the system. This frustration appeared to cluster around time constraints. I felt a large amount of negative emotion directed towards the portfolios.

SOME THOUGHTS TO COMBINE THE TWO

What does some theory have to teach me about the concept of a portfolio system?

Russell (1926: 152) in his essay *On Education* notes that "A considerable part of the working day should be set apart for voluntary self-directed study . . . but the pupil should write an account of what he or she is studying . . . The cleverer the pupil, the less control is required." In their work with the mathematically precocious in the United States, Stanley, Keating and Fox (1974: 18) confirm the need to allow for self-paced work: "Small wonder that even with a class that seems to have been grouped strictly homogeneously some students find the work too easy and others find it too difficult, unless self-pacing study techniques are used."

The portfolio system allows a lot of scope for a back and forth movement where learning co-emerges as a dialogue forms between teacher and pupil. Errors that occur can be effectively "excavated and interrogated" (Davis, 1996: 249) in an enacted process.

The system, however, cannot be imposed and I await the beginning of a new year to see whether the class wishes the system to continue or not. Wilson (1996) relates an experience where he encounters negative energy in a classroom directed at his use of certain video footage. He reacts and later reflects on how to act in the moment in the future to prevent negative emotion of some pupils affecting a positive experience enjoyed by others. The gambit he proposes is to have an imaginary box in class into which pupils can put thoughts of negative emotions. These can then be returned to out of the immediate situation.

I am very positive about the system and would strongly want it to continue, but struggle with balancing the voices in my class. Studies involving a variety of voices show that it is difficult not to privilege one, or a group, of those voices. In research such as this, how does one not particularly privilege one's own voice in conclusions? This is indeed one of the criticisms of enactivism that Fenwick (1999) highlights. It is only through an opening of multiple interpretations that one can guard against this.

FOURTH SET OF REFLECTIONS:
OTHERS BEING BRIGHTER THAN ME

TASK EXERCISE 4:

Imagine that you are teaching a class in which one of the boys writes:

"I feel embarrassed as to how much lower my marks are
compared to some others in the class."

How would you respond to such a statement?

PUPIL REFLECTIONS

Sipho:

I . . . feel embarrassed as to how much lower my marks are compared to some others in the class.

Lin

I have been feeling like the dumb one in my class and I don't pick things up very quickly and everyone else looks like they know exactly what is going on. But like they say, "Don't underestimate your worth by comparing yourself with others. It is because we are different that each one of us is special."

Robyn:

I think that I was under the impression that ad maths would be incredibly difficult and that I was going to be the only person doing so badly.

MY REFLECTIONS AND SOME THOUGHTS TO COMBINE THE TWO

It is a concern of mine that, even in a set of mathematically talented pupils, there is going to be a range of abilities. This could lead to individuals setting up false perceptions of themselves as being "stupid" when in fact their levels of comparison are being unnaturally skewed. What happens to the self-esteem of these pupils is of importance to me. Is a programme that, in all good faith, attempts to offer them something more, negatively affecting them?

From the evidence of the previous sub-collection, it would appear that those at the bottom end of the class who continue the programme, are generally re-affirmed about their mathematical talent by the end of grade 12, that they are not negatively affected, nor, however, are they perceptibly advantaged.

What happens, however, to those who leave the programme? This is a serious concern of mine that I investigate further in the next section.

SUB-COLLECTION 4c
OF MY THOUGHTS AND ACTIONS:
AND THOSE WHO LEAVE

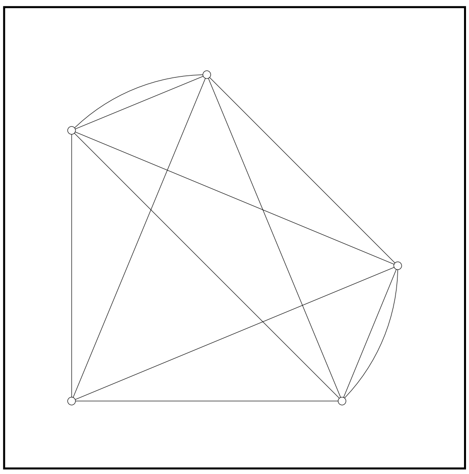
A sub-section in which I consider the effect that an Ad mathematics programme has on those who opt out of the programme prior to the end of the course. I utilise some further statistical tools to see what happens to these pupils and conduct an interview to add to my information.

"Humanity had always been fascinated by the mystery of the falling dice, the turning card, the spin of the pointer. Machines that behaved in a purely random way – events whose outcomes could never be predicted, no matter how much information one had – from these philosopher and gambler could derive equal enjoyment."

(Arthur C. Clarke in *The City and the Stars, 1956*: 48)

"None of them, however, identified the real problem that underlies our crisis of ideas: the fact that most academics subscribe to narrow perceptions of reality which are inadequate for dealing with the major problems of our time."

(Fritjof Capra in *The Turning Point, 1982*: 6)



During the teaching of the Ad mathematics classes, I have recently been concerned about what happens to those who move out of the programme before its completion in grade 12. Has being a part of the programme been beneficial to them? Or do they move out and flounder when returning to higher grade classes?

Jason's comment (current grade 10 class) in a portfolio piece is relevant:

> "One thing that I have picked up is that you either do ad-maths, give it your best, work hard and never give up, or you don't do it at all. I have seen too many people from different ad-maths classes, drop out with the hope that their marks would improve and the exact opposite occurs."

Maturana and Varela (1987) consider the case of a lamb being removed from a mother for a short while after birth (and then being returned), as opposed to those who are permanently left with the mother. The small change has irrevocable, negative and significant consequences on the future of the lamb.

Jason's comment seemed to back up my concern that there is a perception that those moving out of the programme are negatively affected. I needed to search for evidence to investigate this further. Are those who are moved from a normal grade environment to an Ad programme irrevocably and negatively affected?

A small grouping of the grade 9 class from 2001, moved out of the programme for their grade 10 year. This group of 10 children moved into one class accompanied by 25 others who had been through an ordinary mathematics programme for their grade 9 year.

Some of the non-ad group were very competent mathematicians who had been invited to be part of the ad group, but had declined the invitation. This whole grouping therefore provided a controlled background against which to monitor the performances of those who move out of the ad programme.

Figure 5 plots a linear regression of grade 8 results against their grade 10 results for the group that had not had the ad-mathematics experience. (The R^2 value was 0,42.)

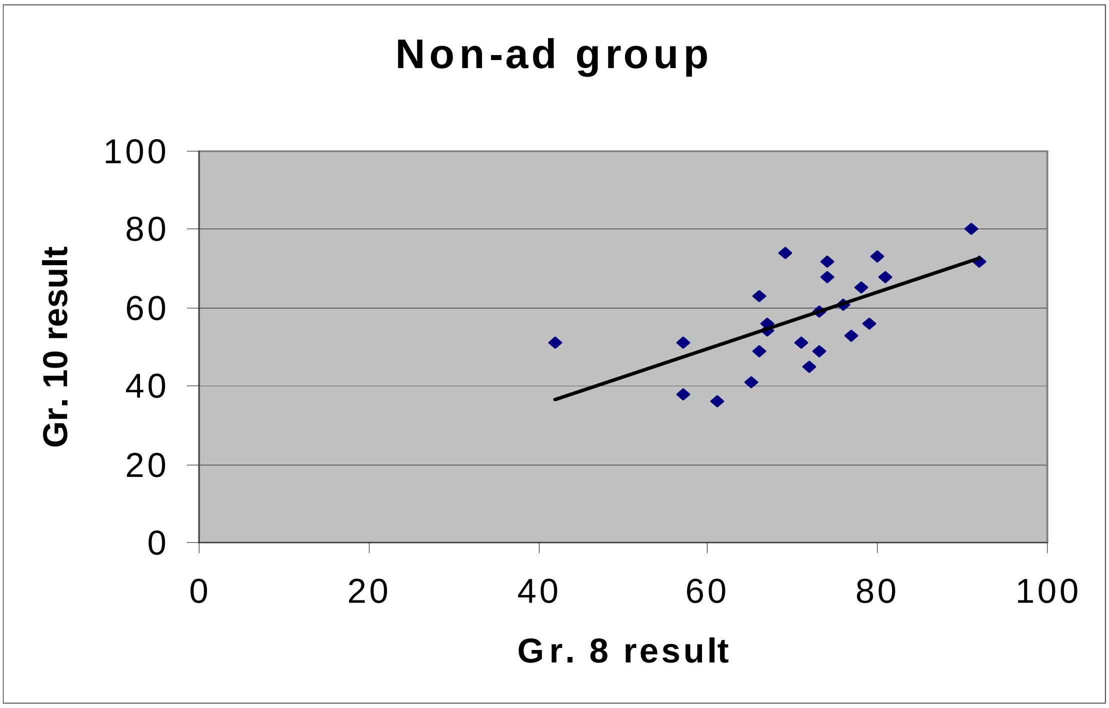


Figure 5

I then super-imposed the results of those who had experienced the ad-mathematics programme for a year and reverted to regular higher grade for grade 10.

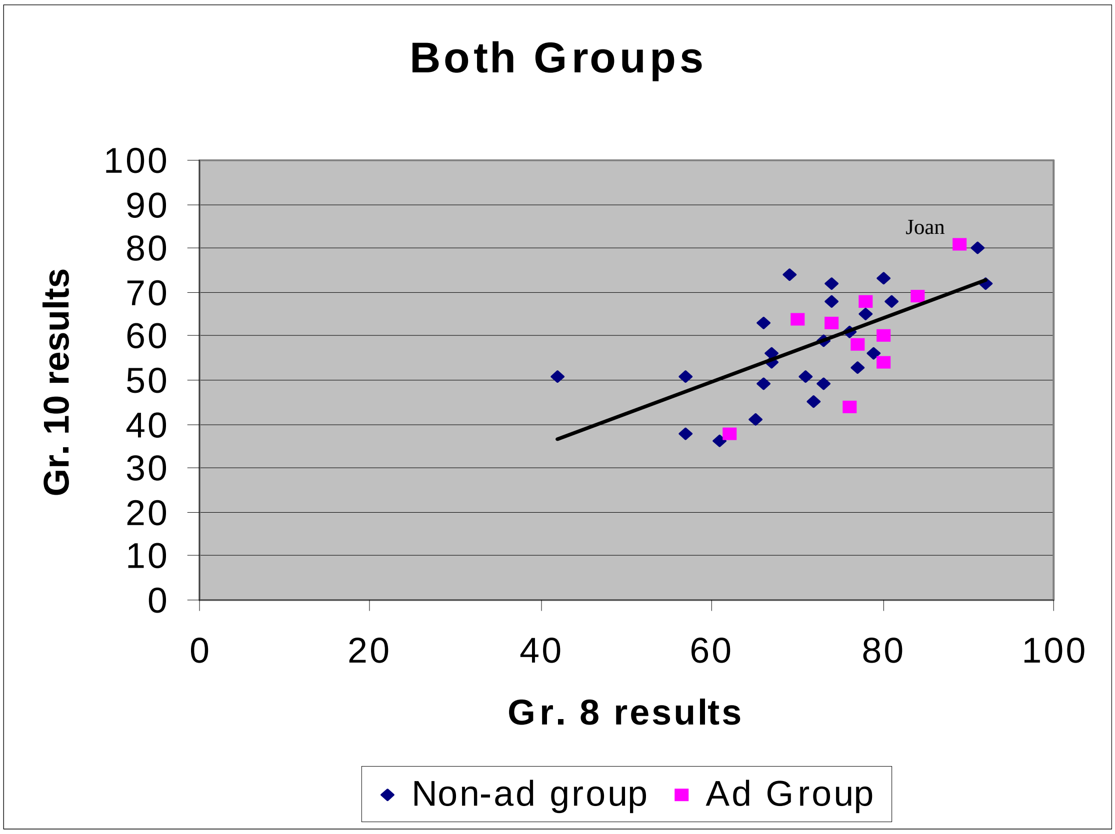


Figure 6

The evidence suggests that 5 of those in the Ad programme were now achieving results better than would be expected, while five were achieving worse results. On average the results moving from grade 8 to grade 10 of the non-Ad group had dropped by 13,5%, while the average of the ad-group indicated a drop of 17,1%.

To test the significance of this change, I then calculated the amount that each pupil's mark had changed from grade 8 to grade 10. I could not assume that these two sets of data were normally distributed and so made use of a non-parametric test.
Stating a null hypothesis that there is no significant difference between the two samples, I performed a Mann Whitney U test on the data. Because of the size of the samples (24 for non-ad group and 10 for ad group) I could not utilise tables to test significance and so used a z approximation that yielded a value of 0,23.

Hence I could not reject the null hypothesis and so conclude that there is no significant evidence to indicate that the programme either negatively or positively affects those who drop out of Ad mathematics.

The statistics, however, open numerous avenues for further questions.

- Why had some gained benefit, while others had not?
- Had their perception of mathematics changed at all?
- Had their perception of themselves and their abilities changed at all?

It has been shown that, as time progresses, intelligence alone is not enough to ensure future success. Terman and Oden (1959) (quoted in Potgieter (2002)) performed a longitudinal study of gifted children and demonstrated how this group succeeded differentially in later life based on differences in a variety of attitudes (now clustered under the umbrella of the label "emotional intelligence"). It is therefore essential that, for an Ad mathematics programme to be successful, it must develop all forms of intelligence.

Is there evidence in these 10 candidates who drop from the Ad mathematics programme, 5 of who succeed and 5 of who do not attain levels one would expect, for other reasons for their performance or lack thereof?

To consider some of these differences I interviewed Joan – the one who appeared to have gained most benefit from the programme after leaving it. Her reflections are still positive about mathematics as a subject and she has good memories of the Ad mathematics class. When asked if she could have her grade 9 year over again whether she would or would not do the Ad programme again, she is sure that she would still choose to do the one year. Her main reasons for leaving were:

- how hard she had to work to attain good marks.
- how stressed she felt at the level of difficulty of the work (more so than stress about the amount of work that was required).
- that she did not feel comfortable being left to study the work on her own.

When asked what could have been done for her to stay in the programme, she included:

- being helped through the mathematics more gradually
- less jumping between different concepts in mathematics

Many of the issues raised in this section need further investigation in follow-up studies:

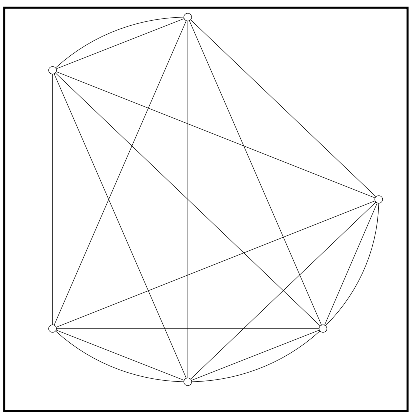
- What continues to happen with these ten up until their grade 12 year?
- What happens with the children who move out of other Ad mathematics programmes not taught by myself?
- Is there normally a portion of the mathematically precocious who drop out, no matter what the system, and whose marks show a significant decline between early and later years?
- What causes such a decline if indeed it exists?

COLLECTION 5
OF MY THOUGHTS AND ACTIONS:
RE-COGNISING MY SPACE IN AN AD PROGRAMME

A section in which I consider what I am as a teacher and how the pupils perceive me. I split the section into two sub-sections in which I focus on two particular aspects of my practice – firstly the perceptions of two different classes of my teaching style and secondly my questioning / listening skills.

"Jeserac [the Teacher] did not care to be disturbed from his ordered way of life, and Khedron [the Jester] represented the unpredictable."

(Arthur C. Clarke in *The City and the Stars, 1956*: 50)



SUB-COLLECTION 5a
OF MY THOUGHTS AND ACTIONS:
TWO ROADS DIVERGED

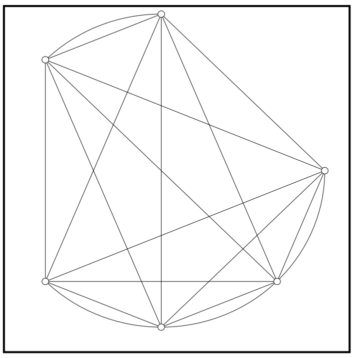
A sub-section in which I consider some difficult parts of my teaching. What different roles do I play as teacher? And what happens when, in certain circumstances, I am forced to overemphasise one of those roles? What gambits can I possibly employ in the future? A difficult section to write, where no research instrument seems to be giving me the "truth" or at least enough of it. And so in the murkiness I return to the safety of numbers

"Something was eluding Alvin, though what it was he did not know. Again and again he tried to fill in the blank spaces, while the instrument read the shifting patterns in his mind and materialised them upon the wall. It was no good. The lines were blurred and uncertain, the colours muddy and dull. If the artist did not know his goal, even the most miraculous of tools could not find it for him.
For his own peace of mind Alvin must return to the tiny, familiar world of Diaspar, seeking its shelter while he came to grips with his dreams and his ambition; the one who had ventured out among the stars was coming home as a frightened child runs back to its mother."

(Arthur C. Clarke in *The City and the Stars, 1956*: 17, 225)

"The Master had never heard him speak so fervently. He walked on in silence for a little, then said: "There is truth, my boy. But the doctrine you desire, absolute, perfect dogma that alone provides wisdom, does not exist. Nor should you long for a perfect doctrine, my friend. Rather, you should long for the perfection of yourself. The deity is within you, not in ideas and books. Truth is lived, not taught.""

(Hermann Hesse in *The Glass Bead Game, 1974*: 80)



My grade 10 Ad class and I were having an informal discussion about school and what was going on at the time, when Maria said to me, "People say that you are nicer outside of school than in school. We don't see a difference, between the two, you're the same to us in our class and outside of class." Tracy responds, "That's because we're special." I paused, because the comment worried me and reinforced my concern that I react differently to different classes and in different situations.

I wanted to see to what extent this perception was shared by others, and to what extent that was affecting my teaching of the various classes. It is often the case that different classes evoke different responses from one as a teacher. Equally often the mathematically more competent evoke more positive responses than those for whom mathematics is fraught with difficulties and fears.

By coming to an understanding of the situation I was hoping to be in a position to make more informed choices of my behaviour patterns in different classes and in different situations. Equally I needed to check the veracity of my perceptions.

I read a description of the above statement made by Maria to each of my classes and then gave each child a questionnaire to complete on their perceptions of my practice. I report here on the results of those questionnaires in two of the classes:

- The grade 10 (15 to 16 years of age) Ad mathematics class (Maria and Tracy are part of this class).
- The grade 11 (16 to 17 years of age) standard grade class (In South Africa those who take mathematics opt for either a higher grade or standard grade mathematics course depending on their ability)

Three questions on my questionnaire asked the pupils to rate their perceptions of me on a 6-point scale, from 1 being very little to 6 being very much. I seldom choose to use a 5-point Likert scale with my pupils, because it is my concern that too often respondents choose a middle option. With a 6-point scale they are forced to make a decision about going above or below the average rating. All responses were submitted anonymously.

The three questions were:

On a scale of 1 (very little) to 6 (very much) please rate how much you think your teacher:

- enjoys mathematics
- enjoys teaching
- enjoys teaching your class

The results (as percentages) from these two classes are tabulated below:

Enjoyment of mathematics							
Rating	1	2	3	4	5	6	Mean
Grade 10	0%	0%	0%	8%	24%	68%	5,4
Grade 11	0%	0%	0%	0%	15%	85%	5,8

Table 1

Table 1 shows that as far as both classes were concerned they perceived me to enjoy mathematics substantially, but the grade 10s were slightly harder on me than the grade 11's. The numbers seem to indicate that my enjoyment of the subject I teach is evident in my classes. This is of course an acknowledged starting point for any competent teacher. Pupils very quickly pick up on a person teaching material about which they are not passionate, and they quickly exploit such situations.

Enjoyment of teaching							
Rating	1	2	3	4	5	6	Mean
Grade 10	0%	0%	0%	32%	48%	20%	4,9
Grade 11	0%	0%	0%	15%	52%	33%	5,2

Table 2

Table 2 shows that again both classes perceive fairly strongly that I enjoy teaching generally, but that my enjoyment of the subject is slightly greater than my enjoyment of teaching of that subject. This was the beginning of my surprise. Again the grade 10 class are slightly harsher in their estimation.

The surprise for me was that it is my feeling that the reverse is true. I enjoy teaching far more than mathematics. My subject, for me, is merely the vehicle through which I get to act out my true enjoyment, which is teaching. I need to consider carefully what it is in my actions in class that creates a stronger perception of me as mathematician than as teacher in my pupils.

Enjoyment of teaching your class							
Rating	1	2	3	4	5	6	Mean
Grade 10	0%	0%	0%	8%	44%	48%	5,4
Grade 11	0%	9%	27%	45%	18%	0%	3,7

Table 3

In table 3, suddenly huge differences begin to appear between the two classes. The grade 10 class, who had been harsher in the previous assessments, came through strongly of the opinion that I enjoy teaching their class. The grade 11's are fairly convinced that there is some problem with my enjoyment of teaching them as a group.

Each child may, of course, have a different perception of what each rating on the scale means to him or her. It is therefore useful for me to look at their individual ratings for each of the previous two questions and see how many of them thought that I enjoyed teaching generally more (or less) than teaching their class. In this way any differences in rating scales is avoided.

	Enjoys teaching generally more than teaching my class	Enjoys teaching generally as much as teaching my class	Enjoys teaching my class more than teaching generally
Grade 10	16%	24%	60%
Grade 11	94%	3%	3%

Table 4

What things I am doing in these two classes to prompt such varying responses?

With regard to the responses of the grade 11 class, I feel resonance with the words of Caroline Smith (2001: 38), "Trust and fear are the key to understanding how people function in social systems such as classrooms. When trust is high relative to fear, the system functions well, but when fear is high relative to trust, breakdown may occur." In the grade 10 (Ad mathematics) class I am able to move my teaching towards the sphere of trust, but in the grade 11 class, where disciplinary issues prevail, I operate more in the sphere of fear. Gibb (1978) structures levels of learning environment quality from the punitive environment through (*inter alia*) benevolent, participative and emergent to a cosmic environment.

These ideas are borne out further by the responses of these two classes to the later questions of the questionnaire. I posed three further questions:

1. Please describe how you generally perceive your mathematics teacher in class.
2. Are there days when he is different to this general perception? If so, please describe him then. What do you think caused the difference?
3. Please describe how you generally perceive your mathematics teacher outside of class.

The difference in responses to these questions began to clarify for me where and how the two classes had developed different perceptions of my teaching them.

The most frequent response that was made by both classes was to do with my ability to adequately explain the mathematics that we covered. This matches with their shared perception of my enjoyment of the subject. Thereafter both groupings showed significant numbers reflecting on their enjoyment of the discussions that were held on subjects that broadened their view of the world (detailed reflection on this has already been covered in sub-collection 4b).

Thereafter the responses took two different forms. The grade 10 Ad class then listed their perception of me as (amongst others) being a teacher who challenged them to think, created an enjoyable classroom environment, was open-minded and enjoyed teaching and what I is doing.

The grade 11 class overwhelmingly noted that they perceived me as strict and serious. A smaller grouping noted my desire to continually be proved right.

When faced with the question of whether I was different outside of the classroom, another surprising split led me to reflect on what is happening within my classroom.

The grade 10's overwhelmingly had an impression of me when outside the classroom (but not interacting with them personally) as quite formal and strict, while the grade 11 class perceived me as more friendly outside the classroom than inside. In the words of Msimi, a grade 11 pupil with whom I interact in the mathematics class and on the rugby field, “I personally think I like my head of rugby more than I like my maths teacher.”

Much time needs to be spent on reflecting on these issues of my functions in the two classes. With my grade 10 class they perceive me foremost as their maths teacher and in a very peripheral sense as an authority figure in the school. Little else comes into our relationship than what happens in the mathematics class. They recognise the disciplinary role that I play in the school, but do not feel that it affects them. Most of these pupils reflected that on certain days I was different in their class, mostly (in their perception) when I was dealing with discipline issues. They mentioned that they could sense these days, “but we try to cheer him up (and usually do!).” (Loren)

With the grade 11 class, I have been a head of their grade for three years, mostly therefore prior to the time when I began to teach them mathematics. In the former role I often have to deal with disciplinary issues, many of them involving members of my particular mathematics class. It is this perception that is coming through strongly in this class.

How does one separate oneself as a teacher? We are required to perform many roles, not all of them pleasant. How does one create an environment in which pupils and teacher are able to accept these different roles and differentiate between them?

Possibly I am trying too hard to separate myself and am trying too hard not to discuss in my mathematics classroom what I am dealing with outside of it. It is possible that an opening up of this by telling my story more in class may well improve the situation.

As a teacher I need to be able to act in the moment in my classroom to make mindful decisions on the environment in which I choose to operate. When required to act out different roles, I need to mindfully shift between roles, not shift out of habit. I need to build a set of gambits for separating and joining the numerous roles. The method of building a shared story of a moment as discussed in the previous collection could, I feel, be a possible starting point.

Ad hoc reflection

Why were these last few pages so difficult to write? Which is the true me? Or is neither? Or are both? Why is it so difficult to open up issues for which there is no clear solution? Why do I still feel that I am right in steps that I have taken with regard to the discipline of the grade 11 class? Why do I feel mentally exhausted? Why, as for Alvin, are "the lines [so] blurred and uncertain, the colours muddy and dull"? Maybe if I do "not know [my] goal, even the most miraculous of tools could not find it."

SUB-COLLECTION 5b

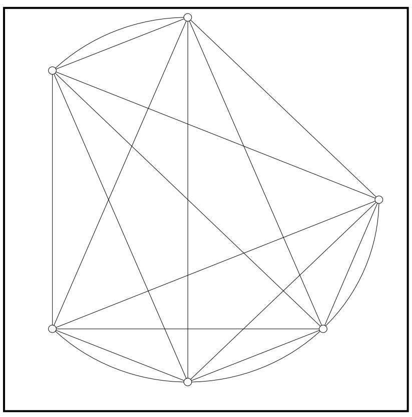
OF MY THOUGHTS AND ACTIONS: LISTENING AND QUESTIONING

A sub-section in which I consider the questioning and listening techniques I use in class. Dialogues happen naturally, but through an increased mindfulness of what is happening, one can take one's natural, intuitive responses and hone them. The rules are there, but are flexible.

"Now listen, Alvin," began Callistron [his friend]. "This is the third time you've interrupted a saga. You broke the sequence yesterday, by wanting to climb out of the Valley of Rainbows. And the day before you upset everything by trying to get back to the Origin in that time-track we were exploring. If you don't keep the rules, you'll have to go by yourself."

"I can't help it," Alvin said a little sulkily. "I think the rules are stupid. I just behave in the way that seems natural."

(Arthur C. Clarke in *The City and the Stars, 1956*: 13)



For a while my listening and questioning technique has been bothering me. I regular wonder why I ask questions of pupils – to check their understanding or to get them to my understanding. I find myself getting tied up in knots, because of a strong current notion of mine that I should not tell pupils, but should try to get them to construct their own knowledge. I am starting to get over the guilty feeling that I used to have when I found myself in telling mode.

Clarke and Lobato (2002: II-16) note that, "It is our contention, however, that the teacher's mathematics can also find legitimate voice in the classroom in the interest of stimulating the development of the student's mathematics." And Davis (1996: 235), similarly comments, "I, the teacher, can still tell; I, the teacher, can still orchestrate. However, it is the learner, and not I, who determines whether I have told or orchestrated." Thus I will only know whether I am telling or not telling when I open discussion and allow open discussion in my classroom. Beyond that I have to learn to listen – without judgement.

After videotaping a number of my lessons, a moment in a lesson with my grade 10 class stood out as worth interrogating in the light of my listening and questioning technique.

Davis (1996) discusses three types of listening. For each I draw a brief transcript from my lesson to illustrate the point.

Evaluative listening:

Where the purpose of the listening is to judge what is being said.

Teacher:	Now it is so tempting to say . . . well 12 km/hr that way and 18 km/hr this way (indicating on diagram on board) . . . the average speed is 15 km/hr. It's very tempting, isn't it?
Class:	Yes.
Teacher:	But that's wrong.

Interpretative listening:

Where the purpose of the listening is to understand what is being said.

Jack:	Why? Because of the wind?
Teacher:	No.
Francois:	Because of the time taken to complete it . . .
Teacher:	What do you mean by time taken to complete it?

Hermeneutic listening:

Where the listening is "concerned with investigating the conditions that make certain understandings possible. It asks not only, What is it that we think? but also, How is it that we have come to think this way? – all with a view toward affecting how we act in the world." (Davis 1996: 18)

Francois:	He is spending most of the time at the average speed . . . No . . . err . . .
Teacher:	Tell me more . . .

In addition to these three ideas, I use the ideas of Clarke and Lobato (2002) to distinguish between telling and not telling in the following interaction.

A LESSON

The class had spent a couple of days working through speed, distance and time word problems that require the use of linear equations in their solution. I had set a number for homework and entered class ready to address any problems on these questions.

We were to start with one involving a cyclist travelling a two-way journey of 36 kilometres each way. On the way out he is faced with a wind and travels at 12 km/hr, while on the way back he has the advantage of the wind and can travel at 18 km/hr. We needed to find his average speed.

T: How much does it help? Is the extra energy you use going into the wind equally repaid going with the wind? Is it equally repaid or do you wear yourself out going into the wind? Is there a break-even point? Is there a maximum point? What is happening?

When Clarke and Lobato (2002) write about when to tell or not to tell, they formulate a model of initiating and eliciting. "If the mathematical idea originates with the teacher, then the teacher is operating in an initiating mode and if the mathematical idea originates with the student(s) then the teacher is operating within an eliciting mode." (2002: II-16) Here I begin the discussion in an initiating mode, posing questions to which I require no answer. They are there to prompt thinking in the pupils about the reality of the situation.

T: So cycling at 12 km/hr into the wind and returning at 18 km/hr, what is the average speed? Now it is so tempting to say . . . well 12 km/hr that way and 18 km/hr this way (indicating on diagram on board) . . . the average speed is 15 km/hr. It's very tempting, isn't it?

Class: Yes.

T: But that's wrong.

Most definitely still an initiating mode response, and an opportunity to bring them to a point of contradiction in their own thinking is absolutely lost by a blunt statement that they are wrong.

Von Glasersfeld (quoted in Clarke and Lobato, 2002: II-19) notes that learning can occur with a change that "takes place when a scheme, instead of producing the expected result, leads to perturbation, and perturbation, in turn, to an accommodation that maintains or re-establishes equilibrium." Here I do not allow the perturbation to manifest itself enough and I restore the equilibrium too quickly.

Class: Why?

Jack: Because of the wind . . .

T: No.

Francois: Well the time taken to complete it . . .

T: What do you mean by the time taken to complete it?

Francois: He is spending most of the time at the average speed . . . err . . .

T: Tell me more . . . (beckoning with hand)

Francois: Um . . .

Francois seldom offers responses in class, but when a practical problem that vaguely has an engineering component to it comes to the fore, he is very keen to participate. Here I think that he understands what the problem is, but is struggling to convert his thoughts into words. He is offering me an opportunity to switch into eliciting mode, to play Hermes and try to understand what he is saying. But I am still in initiating mode – the knowledge and problem lie within my realm and in wanting to get to the answer I turn to Robert who offers to put my thoughts into words. Another opportunity has passed.

Robert:He will be travelling at 18 km/hr faster . . . less . . .

T: What you are saying is he will be travelling at 18 km/hr for a shorter time than he will be travelling 12 km/hr.

Maria: Why?

T: Why? Well, think about it. If he is travelling at 18 km/hr, how long will the journey take him?

Tracy: 2 hours.

Maria: Why? (intensely)

T: Because he is travelling 36 kilometres . . .

Tracy: 36 divided by 18.

Here was an opportunity to test whether Maria did actually understand, but it is passed over, again in a desire to get to the end.

T: . . . and he will be travelling at 12 kilometres for 3 hours. So this journey will take him 3 hours and this journey will take him 2 hours (indicating on diagram). Well, what is speed? Speed is . . .

Class: Distance over time.

T: What is the distance?

Robert: 36.

Others:	No 72.
Teacher:	72 divided by 5 is . . .
Class:	14,4.
T:	14,4 km/hr. So can you see? We expected 15 km/hr, but he was travelling at 12 km/hr for longer. He is travelling for a longer time at a slower speed. Sorry there was a hand somewhere.
Maria:	Don't worry.
Thando:	Why is it 72?
T:	Why is it 72? There and back again (indicating on diagram).
Thando:	OK
T:	Any other queries on that?

The lesson then moved onto generalising the problem for a distance x kilometres and looking at a number of other peripheral issues to do with question 1 of the exercise with which they had been busy.

T:	Alright, ladies and gentlemen going onto number 2.
Tracy:	Isn't 14,4 a very strange average to have?
T:	14,4? What's strange about it?
Tracy:	It's so arbitrary. Although it's probably even more weird if you're going at exactly . . . Never mind what I just said.
T:	Why would that be more weird?

I move into eliciting mode. "Elicitation occurs when the teacher wants to learn more about the student's images, ideas, strategies, conjectures, conceptions, and ways of viewing mathematical situations." (Clarke and Lobato, 2002: II-21) I am employing the hermeneutic method – wanting to find out more about how Tracy understands this problem.

I am, however, concerned with my knowledge too. Once Tracy started speaking of 14,0 being weird in comparison to 14,4, I immediately think back to a perception that I have picked up in pupils that the number 0 is somehow inferior to the other numbers. This relates to work where, when solving an equation where the root is 0, many pupils will write that there is therefore no solution. The number 0 regularly gets omitted in the solution of inequalities too.

The hermeneutic method allows us to "ask not only, What is it that we think? but also, How is it that we have come to think this way?" (Davis, 1996: 18)

I feel that this What – How dichotomy links well with the Initiation – Elicitation dichotomy. I am spending too much time still on the What (and the adjustment of that What to fit my What) and not enough time on truly understanding the How. As Davis (1996: 24), quoting Silverman, notes: "the task of hermeneutics is to raise questions rather than to answer them, to ask about rather than conclude for, and to make a place where positions *can occur* rather than speak *from* positions."

Tracy:	Because it's like too . . .
T:	Is it more weird to go at 14,0 km/hr or 14,4 km/hr?
Maria:	No 14,0 km/hr.
T:	14,4? Alright. Is the chance of getting 14,0 more or less than getting 14,4?

No sooner have I moved into eliciting mode, than I move back to initiating mode.

Class:	More / less.
Loren:	It doesn't really matter, because they don't know . . .
T:	Right, let me ask you. I've got ten balls here. I've got ten balls here. What is the chance of pulling a 0 out? (pause) Oh sorry – the balls are numbered 0 to 9. Alright. I've got ten balls. What is the chance of pulling one . . . err a ball with 0 on it out?
Class:	1 in ten
T:	What is the chance of my pulling a ball with 4 on it out?
Class:	1 in ten.
T:	What is the chance of riding at 14,0 km/ hr? What is the chance of riding at 14,4 km/hr?
Linda:	The same
Maria (to Tracy):	But that's a different question to what you're saying

T:	Why is that a different question to what Tracy was saying?
Maria:	Because Tracy was saying that it is more unusual for you to be riding at a perfectly exact number than to be riding at something comma something. You don't usually ride at exactly 14 km/hr.

Maria gives me the opportunity to move back into eliciting mode, realising that I have misunderstood Tracy. I continue with only a slight modification to explain my line of thinking.

T:	You're asking me - I've got ten balls here again. What is the chance of pulling a 0 out? Alright – one in ten.
Linda:	You cannot get 0.
Class:	Laughter
T:	So what is the chance of pulling a 0 out – one in ten. What is the chance of pulling not a 0 out?
Class:	9 in ten.
T:	9 in ten?
Jack:	9 in ten chance
T:	So is that what you're saying – it's more likely to be comma something than it . . . But would . . . should we be surprised if it is exactly 0?
Maria:	Ja.
Tracy:	But the only thing, I know on the speedometer it doesn't often like say on . . . If you're cycling then it like doesn't like go equally it like jumps so like . . .
Jack:	Ja sir, when you're riding at like 14 and you suddenly go faster there's a big jump . . .
Tracy:	It doesn't like go 14 one two three, it will go 14 four. And . . .
T:	But if you look at it at any point would it be possible to see 14,0?
Tracy:	Ja.
T:	Would it be possible to see 14,4
Tracy:	Ja.

T: Would you be surprised if it was 14,0?

Class: No.

T: Well shouldn't it be one in ten times that you look at it that you would expect to see a perfect number? (Tracy nods.)

Class: (General chatter.)

Tracy: It would be quite fun to cycle and keep it on the same thing.

T: Alright then going onto number 2.

Tracy finally offers me one last opportunity to take the idea from the theoretical to the practical situation. If I was truly open to pupil knowledge this was a time to join mind and body, thought and action in understanding the problem – why not go outside and try it with somebody's bike? But the pressure of time forces me on to number 2.

The essence of my listening must not be to look back on a moment and report that I was or was not listening hermeneutically as I have done here – simple knowledge of hermeneutics will allow me to do that.

The essence could rather be for me to be aware in the moment that I am listening hermeneutically, having mindfully decided to do so. Here I think is the essence of linking theory and practice. The theory can tell me what I am doing, or an 'expert' could point it out to me, but that theory is no good to me, however ardently studied, unless I can capture myself in the moment to apply the theory.

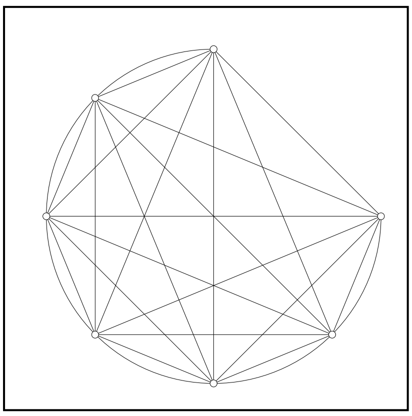
Theory tells me what I **should** have done. I need to act in the moment of my practice to realise what I **could** do (Pimm, 1993).

COLLECTION 6
OF MY THOUGHTS AND ACTIONS:
RE-COGNISNG A SHARED SPACE: A HALCYON DAY

A section in which I consider one of those lessons in which things just seemed to go right. I attempt to use the discipline of noticing to create a learning event for myself and those in the class. In my dissertation I therefore begin to move from the space of the pupil and the teacher to a shared space.

"They were set down gently in a large elliptical chamber, completely surrounded by windows. Through these they could catch tantalising glimpses of gardens ablaze with brilliant flowers. Alystra was enchanted by their beauty."
And yet, even while they baffled him, they aroused within his heart a feeling he had never known before. Alvin had met love in Diaspar, but now he was learning something equally precious. He was learning tenderness."
(Arthur C. Clarke in *The City and the Stars*, *1956*: 33, 106)

"Teaching is a fabric of relationships. It is an identity. These ideas are implicit in the notion of pedagogy, and it thus that an understanding of the special relationship between teachers and learners provides us an opportunity to heal the modernist separation of life and work; it challenges the belief that we can live differently and apart from the way we make our living. It allows us to act on Wendell Berry's warning: "If we do not live where we work, and when we work, we are wasting our lives, and our work too." It is thus that, when I say that I am a teacher, I am not saying how I make my living, but how I live. I am announcing where my work – and therefore my life – take place."
(Davis, 1996: 175)



My grade 10 Ad mathematics class had been learning about the solution of simultaneous equations for a few weeks. I remembered some mathematically rich work that I had discovered a few years previously on the solution of certain families of simultaneous equations. I ask you to consider the task exercise below to allow you entry into the moments described later.

TASK EXERCISE 5:

Solve the following pairs of equations simultaneously:

$2x + 5y = 8$ and $5x + 7y = 9$

$4x + 6y = 8$ and $x + 6y = 11$

Now consider the general case of all pairs of simultaneous equations with coefficients in arithmetic progression.

Now attempt to find what happens in the case of simultaneous equations with coefficients in geometric progression.

I left the lesson wondering exactly what had gone right in the lesson, I sat down that evening and tried to write an 'account-of' the whole lesson. Wondering if the experience for the pupils had been as stimulating as it had been for me, I decided to try to introduce them to the discipline of noticing. Unsure how to get them to give an 'account-of' the lesson without having to go into too much detail, I simply asked them to write the first paragraph below.

And so they wrote two paragraphs for me:

1. If you had been an unseen visitor in yesterday's class, try to record impartially what you witnessed happening in the class.
2. Try to describe your feelings and what you thought during the course of yesterday's lessons.

I then attempted to take our stories and merge them into a shared story of the lesson. It is this that follows. Their 'accounts-of' are in dark blue, their feelings in light blue and my 'account-of' in purple.

As I came into the Ad maths class, I saw a bunch of teenagers, looking forward to the weekend, with maths being the last thing on their minds (Keshan). Walking into Maths on Friday, I was thinking to myself, “oh no, here we go, a double period of maths. “How was I going to survive?” (Keshan). The grade 10 class was about 25 big, a mixture of boys and girls, about 15 or 16 years old. The teacher greeted them at the beginning of the lesson and sat on a stool at the front of the class. The pupils sat in groups of about 6 on chairs at individual tables. The front of the class faced an OHP screen, on the side of the class was a whiteboard.

The teacher began the lesson, one on the solution of simultaneous equations, by asking whether there had been any problems with the homework the night before. A worksheet with a dozen problems had been sent with them the day before. All the equations had been of a linear form with coefficients in arithmetic progression. The pupils expressed happiness with the fact that all the answers had turned out to be the same, -1 and 2. There was, however, general uneasiness about the final question $2x+5y=8$ and $4x+10y=16$. The teacher worked through this on the OHP, drawing the conclusion that any value of x and its corresponding value of y would satisfy the equations, one of these sets of solutions being -1 and 2. The final statement of the worksheet had asked the pupils whether there was anything that they had noticed about their solutions and could they prove that for equations of this type, this would always be true?

All said that they had noticed that the solutions were all the same. One girl mentioned that she had noticed that the difference between the coefficients had been the same in both equations, but this had only been true for some equations, in particular the first one. Others then said that a more general thing had been that in any one equation the difference between the coefficients was the same. Pupils then looked down at the questions again and some contested that this was not true for some of the latter questions where negative coefficients appeared, others responded that it was still true, because the same number had been subtracted, not added in this case. A similar argument happened about some of the ones with fractions. The teacher and others responded that if they checked the fraction work, it held here too.

At this point, one of the boys made the comment that all the coefficients were in arithmetic progression. There was a brief moment of silence , the teacher asked how he knew this, he responded that he had read it in the UCT Mathematics Digest a little while ago. There was a cheer from everyone and he took a bow. Out of the blue someone called out, arithmetic progression. The teacher looked astonished and asked where he had heard this, he replied, "the Maths Digest . . ." and the whole class erupted in applause (Keshan).

The teacher asked whether anyone had managed to prove the result. He then went to the whiteboard and said that we need to make a hypothesis about our findings, and proposed, "The solutions to two simultaneous equations with coefficients in arithmetic progression will always be -1 and 2." He then asked them to refine this, because problems might arise. A comment came that the equations had to be linear. The teacher then suggested $3x+4y=5$ and $5y+7x=9$. Although this satisfied the current hypothesis, the solutions were not correct. On a proposal from the teacher the statement was refined to equations of the form $ax+by=c$.

He then asked anyone to try to start the proof. Tracy came up, started off with $ax+by=c$ and $dx+ey=f$. With some prompting, the equations were refined to $ax+(a+g)y=a+2g$ and $dx+(d+t)y=d+2t$. The teacher then asked the class to try to finish off the proof. When I looked at the two equations on the board I thought I would never get it right. (Jason) The lesson started off normally for me, when we first got the equation I was scared, as I often am about Ad maths. (Maria) The teacher kept making statements about the fact that this was great mathematics.

They started quickly, in silence. I saw a class given a difficult hypothesis to prove. I saw some groan, some become dead serious, and some laugh the whole thing off. I was surprised as dead silence crept into the class and learners turned intently to their books. (Jaclyn) I think people would have seen learners hard at work and maybe thought that the teacher had told us to be silent. (Maria) The noise then steadily rose with children working within their groups. Little peaks of noise happened as people felt they were getting near to something. Here sits a class of 25 kids; all working fairly hard on something. And while there's definitely an atmosphere of learning, there's still a sense of enjoyment. Kids are sitting and discussing their work and talking and laughing - but the class is focused and still pretty quiet. The arguments and questions about the work, and the shouts of triumph every now and then would definitely have brought a smile to any person's face. (Robyn) Everyone was working, but everyone was talking and laughing at the same time. (Candice) When there was noise, it was a genuine working noise. (Tammy) Occasionally somebody closed their book with a bang and sat up with a triumphant smirk on their face. (Jaclyn)

The teacher sat at his table at the back. Maria leaned back to say she could not solve the equation she had reached. The teacher asked how she would normally solve a linear equation. She replied that she would get x's to the one side and numbers to the other. She did this quickly and then said that she was still lost. The teacher asked whether what she had could be factorised. She turned back to her group and a few minutes later there was a huge cry of success from that group.

At this point people were moving around the class between groups to see whether others had something that they were missing. During the lesson, I felt mainly frustrated, as each time I tried to work out the equation, I would come out with "$0 = 0$". But, towards the end of the lesson a friend showed me where I had made a mistake, I started feeling a lot more enthusiastic about the equation. (Candice) The nice thing was that we all knew different parts and could help each other out. (Tammy) At this stage everybody was whispering at my table and it was very tempting to start discussing the problem with them. But I wanted this to be *my* proof alone, and I didn't want anybody's help. (Jaclyn) I had to help some of my friends and felt accomplished when I could . . . allow others to understand it. (Jill)

One group appeared to be hung up on something. One of them approached the teacher, because they had reached a point where their equation was of the form $(y-2)(at-dg)=0$. They could see that $y=2$ was an obvious solution, but could not explain why $at-dg=0$ could be ignored as the other possible solution. The teacher did not have an answer and sent them back to work on it, while he worked on the board. I probably learnt more from struggling and then getting the answer than it just getting given to me. (Tammy)

My frustration was growing quickly. Then I tried another route, but got stuck, so I asked people and they had no idea. The bell rang and I was really upset I hadn't got it, so I carried on through some of the second lesson and while everyone was working on the [next] worksheet I got it. I was very happy once I got it. (Julie) When we were given a break at the end of the [first] period my table in the back corner was still full, but you would have noticed that in the corridor the work was still being explained to those who still did not understand. (Bianca) As the bell rang I noticed another strange phenomenon - people actually staying in the class and working, even though they could've been outside with their friends. (Jaclyn)

A little later when all groups had reached some form of satisfaction with their answer, the teacher returned to the front and congratulated them on the way in which they had worked on the problem and returned to the problem of $at - dg = 0$ that had been brought to him. He asked whether anyone had a possible explanation. Nobody had one. He then asked them to relate it to the final question on the worksheet and gave them some hints about it.

Finally he took them back to the hypothesis on the board and asked how they could change it and create for themselves another piece of mathematics that would be their own discovery. The what-if's that were proposed included taking three simultaneous equations and taking non-linear equations. He suggested that they might wish to pursue those on their own. He then asked the boy who had mentioned arithmetic progressions, whether he had read of any other type of progression. He had not. The teacher said that if arithmetic progressions were ones in which numbers were added to the previous term, what other types could exist? The proposal was that numbers could be multiplied by a fixed amount. The teacher introduced the concept of a geometric progression and handed out a second worksheet dealing with this type, also requiring generalisations of results.

The first lesson on Friday, was probably the funnest and most productive lesson for me, since I've been in the ad maths class

(Keshan) I didn’t feel anything terribly strong during the lesson except some irritation and amusement at myself. (Bianca) It was the most exciting thing and very rewarding to get out an answer. It made me realise that this year isn’t going to be as scary as I thought. (Maria) I think I’m on the verge of learning something new, but I’ve not figured it out yet, but I’m on my way to figuring it out. (Thando) Mathematics is art when it’s so complete and just so there. (Linda L) It was wonderful and I suppose I can really say that cause we understood. Things are different when you don’t. (Tracy)

I left the scene satisfied that learning, for these children, is *good.* (Jaclyn)

Whenever re-reading this account I gain immediate re-entry to the environment that existed in the class on that day. I wish now to discuss certain issues that arise for me from such a consideration of a lesson.

Davis (1996: 96 – 97) lists a number of features of a mathematical task:

1. Be all-at-once (counting, locating, measuring, designing, playing and explaining.)
2. Have variable entry levels.
3. Be rich (diverse, unexpected and extensive)
4. Be open-ended.
5. Require more than one mode of reasoning.
6. Include opportunities for independent and collective study.
7. Allow emotional inquiry.
8. Serve as a focal point for joint action.

We can find evidence in the account of the lesson for most of these being features of the task given.

1. The first issue of being all-at-once is not completely covered by this task, as it is of a pure mathematics nature. There is evidence though of pupils needing to utilise a variety of techniques and of thinking strategies for the solution of the problem and for playing and explaining while they are working (lines 82 – 86, 92 – 99).
2. The task has variable entry levels – those who simply solved the equations were honing essential skills, while Paul (lines 114 – 118) was considering fundamental issues about the solution of equations that others had simply ignored.
3. The task is rich – the engagement of the pupils with what they are doing is, for me, the greatest evidence of this richness (lines 71 – 89).
4. The task led to further opportunities – one pursued in class and the others left to the pupils in their own time (lines 143 – 157). Paul continued studying one of the what-ifs for the next nine months, adding piece after piece to his portfolio.
5. The task does not require more than one mode of reasoning – it is predominantly situated within a system of logico-deductive thought processes.
6. Most pupils opted to start the problem on their own (lines 71 – 77), but as time passed and they hit hassles there was a natural move to pooling their resources in

an attempt to solve it (lines 77 – 89, 100 – 102), although some wanted to try to complete the problem on their own (109-111).

7. This problem is ample evidence for the enjoyment that can be derived from a pure mathematics problem. The lows of bafflement and highs of triumph are recorded in lines 78 – 79, 84 – 86, 102 – 103 and 161 – 164.
8. The movement in class between groups and outside into the passage led to a notion of a unified goal – one that took most members of the class along with it (lines 100 – 102 and 128 – 131).

Further items open up questions about my practice:

The defining moment for me in the lesson – the point at which I was motivated to take an ordinary lesson (double period at the end of a Friday!) and give it a chance to be something special was when Keshan made his comment about arithmetic progressions (lines 41 – 49). I had been trying to get them to read mathematical material, particularly the *U.C.T. Mathematics Digest*, for a long while. Here was evidence that I had been partially successful. Chaos theory explains the possibility of small occurrences having ramifications of a far greater magnitude than the original perturbation. In that moment a small incident built into an ever-expanding spiral. It is interesting to see that Keshan, in his reflections shares this moment with me (lines 158 – 160) – a serendipitous coming together that builds into something for everyone. It is also noteworthy that up to this point (line 46), hardly any pupils have recorded anything in their reflections of the lesson – nothing is standing out for them – it is only my voice that is heard describing what has happened.

The comments of Jason and Maria (lines 65 – 68) remind me of Jaclyn’s reflections about her struggling and then overcoming problems in collection 4b on page 44. It is important in my teaching to try to get the pupils to achieve closure on problems.

In line 92 I choose to sit at my table and not participate in the discussions that are happening unless specific issues are brought to me. This is a conscious decision, because I am afraid of spoiling the moment of discovery for the pupils. In making this choice, however, I also cut myself off from noticing discussions in which people are too lost in the problem to even engage with it. Too often I remove myself in this manner and need to consider gambits to engage myself more fully in the discussions without being tempted to give too much of my knowledge. I need to develop my ability to question and listen hermeneutically.

The co-owned story that has emerged offers me many avenues of reflection and would be a valuable tool to utilise in class on a regular basis for isolating snapshots of incidents that occurred in class. It is something I particularly wish to return to as a gambit when considering the issues presented about my grade 11 standard grade class in collection 5a.

COLLECTION 7
OF MY THOUGHTS AND ACTIONS:
A PAUSE ON THE WAY ON

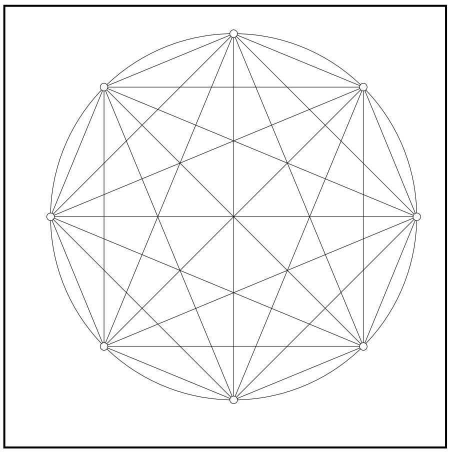
A section in which I try to draw the threads together of what I have learnt and in which I attempt to put together a theory of what I **could** do to improve my practice.

"This was the moment of choice. Until this instant, he had always been able to turn back if he wished. But if he stepped inside that welcoming door, he knew what would happen, though not where it would lead. He would no longer be in control of his own destiny, but would have placed himself in the keeping of unknown forces."
(Arthur C. Clarke in *The City and the Stars, 1956*: 84)

"The changes that result from the interaction between the living being and its environment are brought about by the disturbing agent but determined by the structure of the disturbed system."
(Maturana and Varela, 1987)

"Learning should not be understood in terms of a sequence of action, but in terms of an ongoing structural dance – a complex choreography – of events which, even in retrospect, cannot be fully disentangled and understood, let alone reproduced."
(Davis, Sumara and Kieren, 1996: 153)

"Thus his path had been a circle, or an ellipse or spiral or whatever, but certainly not straight; straight lines evidently belonged to geometry, not to nature and life."
(Hermann Hesse in *The Glass Bead Game, 1974*: 352)



It is a time to pause along the path that has been laid in the walking, in the space that has been built in the practice and reflect on what has happened during the course of my research.

It has been a rich, but frustrating time. With an increased awareness of the possibilities has also arrived an increased frustration with catching myself after the moment, having done something that I no longer wish to have as part of my practice. This resonates so strongly with Davis and Sumara's enactivist study in a small elementary school: "One of the most frustrating aspects of [our] experiences was the tendency each of us had to fall back on teaching behaviours that we felt were incompatible with (and even contradictory to) the enactivist theory of learning and understanding." (Davis and Sumara, 1997: 113) Effective change is, however, a slow process and as much as red and orange can be easily differentiated in their extreme forms, so also is there a spectrum in between through which one must move before one becomes the other.

And so I now can choose whether I do or do not wish to learn from what I have found - to choose whether to move with the perturbations that have been caused, or to ignore the lessons. Maturana and Varela (1987: 245), however, suggest that action is inevitable once one knows that one knows. "The *knowledge of knowledge compels . . .* It compels us to realize that the world everyone sees is not *the* world but *a* world which we bring forth with others. It compels us to see that the world will be different only if we live differently."

It is through this study of my thinking and actions that I hope to build a mindfulness of what has happened in the past and be equipped with the gambits described by Mason to make mindful decisions in my future Ad mathematics programmes. "Through mindfulness [I] can begin to interrupt automatic patterns of conditional behaviour." (Varela, Thompson and Rosch, 1991: 122)

My choice is about moving from what the theory tells me I should do to a mindful awareness of my own practice and the wealth of coulds that present themselves to me in every moment in which I capture myself.

It is difficult to take one's practice and build a theory from this. There is the great danger expressed by Varela and Shear (1999: 13): "How do you know that by exploring a method you are not, in fact, deforming or even creating what you experience." My theory needs to be based on my evidence, the validity of which is spoken for by a combination of shared experience and statistical data.

Whitehead (1989: 5) backs this concept of personal validation

> "In grounding my epistemology in Personal Knowledge I am conscious that I have taken a decision to understand the world from my own point of view, as a person claiming originality and exercising his personal judgement responsibly with universal intent. This commitment determines the nature of the unit of appraisal in my claim to knowledge. The unit is the individual's claim to know his or her own educational development."

I feel that it is an interesting space that I have built – a space in which there is potential for me to significantly alter how I learn and teach. I took a step outside of current research methodologies, acknowledging at the outset that my dissertation is as much about my contesting current research paradigms as it is about understanding my practice in an Ad mathematics context.

It was an aim of my research to utilise appropriate tools to answer particular questions. These tools have been drawn from a variety of paradigms and I feel that the blend of quantities and qualities has opened a fuller picture for me than a limited use of one or the other.

Some lessons that I have learned and questions that have been opened include:

- ♦ The top and middle achievers in an Ad mathematics programme appear to have been positively influenced by the programme, both in terms of the marks that they achieve at the end of their schooling and in terms of the higher order thinking skills that they have developed.

- The evidence for those at the bottom end of the Ad mathematics class is inconclusive. They do not appear to have been prejudiced by the programme, nor do they seem to have been positively affected in terms of their marks.
- There does not appear to be evidence to support my suspicions that those who leave the Ad mathematics programmes are negatively influenced. This is, however, a grouping to which I could pay different attention in my teaching of future Ad mathematics classes. The method of hermeneutics may help us to more effectively confront self-esteem issues and feelings of stress or lack of understanding.
- For myself the Ad mathematics programme appears to create a space in which I teach in a different manner, a space in which I allow my humanness to present itself more freely. I could consider the lessons I learn in this space and try to effect similar spaces in other classes.
- What happens to all of these pupils beyond school? Have they benefited? Have my goals of an Ad programme given earlier been realised? Where are they? What are they? Who are they? How are they?

This is a pause for reflection in my space and if my dissertation and reasons for attempting further formal study have been honest and open, then the paths must continue from here. To stop at this point would throw me back into the acquisition metaphor of Sfard. What interests me still? What questions do I still need to answer? What questions do I still need to ask?

- Undoubtedly I feel that I have not nearly exhausted the lessons that the statistical data of pre- and post-test results have to offer. Data from similar programmes could yield a gamut of potential effects of Ad programmes and I have the sense that a whole theory could be built on this basis.
- Another path that presents itself is an exploration of other teachers' experiences in an Ad mathematics programme. With the data that I have developed in this dissertation, I feel that I am now in a position to further explore resonance with others using some of the task exercises developed here.
- Within my practice, I feel that the building of the shared story in the collection on a halcyon day, offers me with a gambit for opening up voices within my

classroom (both Ad and non-Ad groupings) that will begin to balance more effectively whose voice is privileged.

- Whether the Ad mathematics programme has had a wider influence on participants than in the mathematics field has not been addressed in my study. Whether or not there has been significant impact on other subjects could be a source of further information.
- A study of those who are now in tertiary institutions and in the work place would add another dimension that is completely ignored by my current study. I have a sense that there are lessons here as yet untold which will more effectively identify the lessons learnt in Ad mathematics programmes than a mathematical school leaving certificate examination. It would be interesting to see whether or not significant numbers of participants have moved into the academic field.

The stories and reflections I have looked at, so often looked plain to me and what everybody else knew. Although I might have known it, I did not know it about me. Now I tell the story and struggle to find the voice with which to tell it. As Maxine Greene notes (in Jipson and Paley, 1997: 6) that we have "to shape our narratives in ways that do not duplicate other narratives." But when I use my words, they seem plain and simple and not part of this great edifice of the Academy. But I have attempted to follow the advice of Breen (2001: 2) in believing that it is "the practice [that] has to be foregrounded rather the theoretical imperatives of the academy." And I get strength from daredevil research that allows for my expressions to be "everyday and practical, not foundational and eternal." (Jipson and Paley, 1997: 9)

And so if the point of hermeneutics is to "interrupt our unquestioned patterns of acting" (Davis, 1996:20) this has been a journey along with Hermes (the trickster) and Khedron (the jester). The journey has been a difficult one – so often I have wanted to take the path of least resistance – follow the advice I was given. Find a small question, find some numbers describing it, write it up and submit.

That would get me a degree, but it would be lifeless.

How much more difficult was this wanting to unpack the whole story of my teaching, to come to terms with what I am, what I am becoming and what I might be – how much more difficult, but how much more life-full.

BRIEF POSTSCRIPT

After writing up the majority of my dissertation during a long summer break, I have now returned to my classroom and to my practice. And I feel awfully dissatisfied, as one who was blind, but idealistic, who now can see both good and evil within their sight.

For the first time I truly understand the meaning of Maturana and Varela's comment (1987) which I now consider a warning to those who do not really wish to improve their practice: That the knowledge of knowledge compels us to act. This dissertation will forever be evidence for me that I know some things about my practice and about the incredible complexity of the relationships within my classroom.

But acting, although I feel compelled to do so, is so difficult when mind and body, individually knowing the theory, find it hard to put it into practice as mind and body, rather than mind or body.

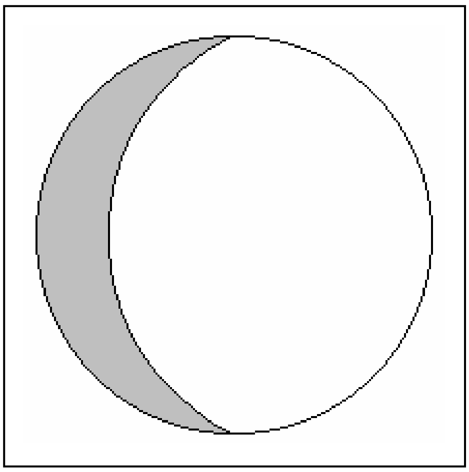
But that is another story . . . . .

COLLECTION 8
OF OTHERS’ THOUGHTS AND ACTIONS:
REFERENCES AND ACKNOWLEDGEMENTS:

A section in which I acknowledge all those who have added to my knowledge, my understanding and my danger. A thank-you to them for their courage in their time and place and for creating spaces in which I could live. Time only will tell which will be left and which will fade.

“I am tempted to say that all knowledge is valuable, and it cannot be denied that you have added much to our knowledge. But you have added much to our dangers, and in the long run which will be more important?”
(Arthur C. Clarke in *The City and the Stars, 1956*: 159)

“I have gathered a posy of other men’s flowers, and only the thread that binds them is my own.”
(Michel de Montaigne)



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My very sincere thanks.

"The adventure was over. Soon, as always happened, they would be home, and all the wonder, the terror, and the excitement would be behind them. They were tired and content."

(Arthur C. Clarke in *The City and the Stars, 1956*: 12)

"There is a special sadness in achievement, in the knowledge that a long-desired goal has been attained at last, and that life must now be shaped towards new ends."

(Arthur C. Clarke in *The City and the Stars, 1956*: 249)